A Theory of Non-Democratic Redistribution and Public Good Provision[*]

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Abstract

This paper proposes a new theoretical definition of (non-)democracy based on two "political rights" parameters (η, k) that capture the extensive and intensive margin of the population's ability to replace the incumbent; and an "individual right/civil liberties" parameter λ that captures the degree to which individual citizens are protected from being rewarded or punished based on their past political activity. The policy space features a trade-off between redistribution and public good provision. I study two types of public good: one that delivers egalitarian benefits, the other that delivers non-egalitarian benefits.

I find that in any regime where the protection of individual rights is not absolute there is a politico-economic force driving toward equal treatment, but this force is tempered if political rights are weak. Regimes with strong political rights and imperfect protection of individual rights turn out to be the most conducive to equal treatment, and they provide egalitarian public goods efficiently. Regimes where individual rights are perfectly protected give politicians incentives to treat citizens inequitably for political advantage; these regimes provide non-egalitarian public goods efficiently.

This model matches a wide variety of phenomena. First, within regimes with strong political rights, variation in λ captures the distinction between pluralist (US)

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and consensual (EU) democracies. Second, the model matches the fact that the US has long excelled at innovative but relatively non-egalitarian growth, as compared with EU democracies after WWII that have excelled at providing relatively equitable but arguably less innovative growth. Third, the model matches the observation that some non-democracies have provided strong "catch-up" growth starting from a very low base, but have struggled with generating original innovation. Fourth, the model allows for non-democracies that are purely extractive and have no redeeming features.

1 Introduction

A lot of the world's GDP is produced by non-democracies (see Figure 1). Yet, little is known theoretically about the political incentives that shape economic policy in non-democracies, and how these might compare to democracies. I address these two questions here.

To address these questions it helps to have an operational definition of non-democracy that nests democracy as a special case. I define a *regime type* based on three parameters: the fraction $\eta \in [0, 1]$ of the citizens who have *political agency* meaning that they can, through their collective action, replace the incumbent politician; the cost k of taking political action against the incumbent; and a parameter $\lambda \in [0, 1]$ which captures civil liberties, specifically, protection of assets from political persecution. When $(\eta, k, \lambda) = (\approx 0, \infty, 0)$ only a vanishing fraction of the citizens have the power to replace the incumbent, supporting a challenger is infinitely costly, and a citizen who mistakenly "backed the wrong horse" has all his/her welfare (job, home, etc) taken away. A liberal democracy is a regime $(\eta, k, \lambda) = (1, 0, 1)$ where all the citizens are able to vote, the cost of supporting the challenger instead of the incumbent is zero, and a citizen who happens to support the losing candidate suffers no material consequences. The triple (η, k, λ) captures the main criteria used by rankings such as "Polity IV" or *The Economist's* "Democracy Index," to classify political regimes.[1]

Given any triple (η, k, λ), I posit that two office-motivated politicians compete for power by making promises to voters (or voter groups). After observing both politicians' promises, every voter (or group) who has political agency decides whether to support the incumbent or, at a cost k, support the challenger. If enough citizens support the challenger, the incumbent is replaced.

[1] The parameters η and k capture the citizens' ability to vote on alternative leaders, whereas λ captures a combination of independent judiciary and civil liberties.

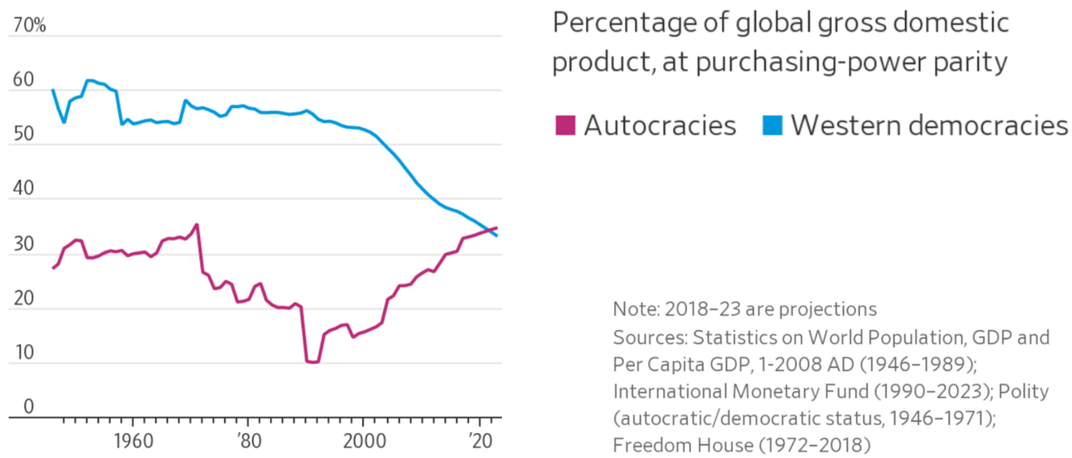

Figure 1: The Growing Economic Power of Autocracies. Reproduced from Foa and Mounk (2019).

The model's description is completed by specifying the policy space – what politicians can promise. Politicians can promise to tax and redistribute across individuals or groups. Alternatively, politicians can promise to use the tax revenue to pay for public goods, which are policies that may yield higher social welfare than redistribution.

The main question I address is the following: given the choice between socially inefficient redistribution and a socially efficient public good, what incentives do politicians have to promise the efficient policy? Obviously, the answer will depend on the specific values (η, k, λ). In addition, the answer will turn out to depend on whether the public good benefits all citizens equally, or whether it creates inequality among citizens. To build intuition, I describe the mechanics of the model next.

Every citizen (or group) i is promised a certain welfare level by either politician. Suppose a politician promises X_i to citizen i. Whenever $\lambda < 1$, that is, the regime is not fully liberal, that promise generates the following incentive to support the politician: "if you support me and I get (or keep) power I will give you the amount X_i, but if you supported my opponent you will only get λX_i from me." The incentives provided by the two politicians' promises are in direct conflict; furthermore, given a sufficiently high probability that either politician wins power, citizen i should honor that politician's incentive *even if* X_i *is small*, and disregard the other's. Which politician wins power, in turn, depends on how many citizens support either politician. Therefore, as soon as citizens receive promises from both politicians, they are locked in a coordination game.

The idea that politicians "compete for coordination" using redistributive politics is new in the political economy literature, to my knowledge.

I will resolve the equilibrium multiplicity in this coordination game using a "global game" approach, and then ask: what kind of promises would incumbent and challenger make in equilibrium as they try to "win the coordination game"? In particular, is there a tendency for promises to be the same for all i?

I find that, when $\lambda < 1$, politicians have incentives to treat citizens equally. This means the following. Suppose a politician promises every citizen \$1, meaning "\$1 if you support me, and \$$\lambda$ if you don't;" then every citizen has the same incentive $(1-\lambda)$ to support the politician. If, instead, the politician promises \$2 to half the citizens and zero to the other half, then half the citizens will have a strong incentive $2(1-\lambda)$ to support the politician, and the other will have zero incentive. It turns out that, in the global game, this kind of unequal incentivization is not helpful: the incentives for citizens to coordinate on supporting a politician are maximized if the politician treats citizens equally. This finding, which is not obvious and is new in the political economy literature, implies that any non-liberal regime $(\lambda < 1)$ pushes both politicians toward egalitarian treatment of citizens.

This force toward equal treatment is not absolute: first, citizens who do not have political agency will be neglected by politicians, and this is a force against equal treatment. But even citizens who have agency will be treated unequally by the challenger when k is large. This is because incentivizing a citizen to support the challenger requires a promise of at least k (otherwise it is a dominant strategy to support the incumbent), and when k is large the challenger cannot afford such large promises to all citizens. Instead, the challenger is better off concentrating his relatively limited resources on a few citizens, which leads to inequitable treatment.

In sum, in any regime with $\lambda < 1$ there is a politico-economic force driving toward equal treatment, but that force is tempered if η is small and k is large. Regimes with $(\eta, k, \lambda) = (1, 0, <1)$ are the most conducive to equal treatment; I call this class of regimes "quasi-liberal democracies" because $\lambda < 1$.[2] The limit point of this class is $(1, 0, 1)$, a regime with special significance because it is the game that is studied in most voting theory; I call it "liberal democracy." I show that liberal democracy behaves differently from any quasi-liberal democracy because when $\lambda = 1$ citizens do not face a coordination game, but rather a dominant strategy game. As a result, the objective function of politicians is qualitatively

[2] It may sound strange to call a democracy "quasi-liberal." In Sections 9.1 and 9.2 I discuss why many democracies are in fact not fully liberal in the sense that $\lambda < 1$, and tie my definition of quasi-liberal democracy back to the notion of corporatist/consensual/coordinated democracy from political science.

different, and it gives politicians electoral incentives to treat citizens inequitably. This qualitative "phase change" is one of the main conceptual insights from this paper.

Having understood that non-liberal regimes (including particularly quasi-liberal democracies) have a built-in tendency toward egalitarianism, but liberal democracy has a tendency toward non-egalitarian treatment, it becomes intuitive that quasi-liberal democracies are the best regime for efficient provision of egalitarian public goods, and liberal democracy is the best regime for efficient provision of non-egalitarian public goods. This is the main applied result in this paper.

In Section 9 I argue that the model's implications are consistent with a number of stylized facts, including: that liberal democracies excel at fostering technological innovation, but authoritarian regimes struggle at it; that some authoritarian states can do well in the provision of broad-based public good such as education, health care, and womens' rights; that other authoritarian states are purely predatory; and that so-called corporatist/consensual/coordination democracies are more capable of adopting strong industrial policy compared with pluralist democracies such as the US.

1.1 Related literature

I limit this literature review to theoretical papers that study policy determination in non-democracies. The most related papers are those where the policy space features a trade-off between redistribution and public good provision; the least related are those where the policy space does not feature this trade-off.

McGuire and Olson (1996) model a redistributive democracy that underprovides an egalitarian public good, and compare it to an autocracy that optimally provides it. Their policy space, like mine, features a trade-off between redistribution and public good provision. The conceptual difference lies in the definition of autocracy: McGuire and Olson (1996) assume that the autocrat is a consumption-motivated "stationary bandit" who faces no competition for power. This autocrat owns all the tax receipts, pays for the public good out of tax receipts, and maximizes the leftover, i.e., tax receipts net of public good expenditures. In their setting the tax rate is set independently of public good provision, which means that the autocrat's payoff equals a fixed fraction of social welfare, in turn leading to efficient public good provision. In my theory, in contrast, the incumbent autocrat acts under pressure of replacement by a challenger. The incumbent (respectively, challenger) promises the public good if and only if it reduces (resp., increases) the probability of replacement. The policy that is enacted depends on which of the two politicians prevails, and it need not be efficient. In sum, the two papers are very different, but they share a focus on the tradeoff between targetability of redistribution and efficiency of the public good, and the notion that democracy provides incentives for inefficient targeted redistribution.[3]

Bueno de Mesquita et al. (2002) also study a trade-off between redistribution and public good provision. They classify regimes according to the size of the selectorate S – the set of people who have an institutional say in choosing leaders – and the size of the winning coalition W – the minimal number of selectors whose support the incumbent needs to remain in power. They find that, the larger the ratio W/S, the greater is public good provision. The logic is that only citizens in W need to be bribed in order for the incumbent to survive, and the public good "wastes bribes" unnecessarily on selectors outside of W. The same logic drives the result in this paper that public good provision is increasing in η (refer to Proposition 2 part 5), but the connection is not perfect because in this paper membership in H is exogenous, whereas in Bueno de Mesquita et al. (2002) the membership of W is determined by the autocrat.

Besley and Kudamatsu's (2007) policy space also features redistribution and public good provision. However, crucially, there is no direct trade-off between redistribution and public good because the latter doesn't cost money to provide. Rather, it is a choice between different policy options that is driven by the politicians' personal preference and that, in equilibrium, signals the politician's type to voters. There are many other differences between my model and Besley and Kudamatsu's (2007), including the fact that they only have two groups of voters, whereas I have a continuum. But there are also some common elements including the fact that the incumbent risks being removed from office and that democracy obtains for certain parameter values of the model. Roemer (1985) studies a sequential game between two office-motivated politicians, an incumbent and a challenger. The focus is on whether the challenger's promises are more egalitarian than the incumbent's. There are many differences with the present paper, including that the incumbent's promises are fixed exogenously. Perhaps the biggest difference is that the policy space is purely redistributive – in my language, there is no public good.

The following papers are interesting models of non-democracy, but their policy space is not "redistribution vs public good," or even "redistribution." Padro i Miquel (2007) highlights that part of the costs for citizens of overthrowing an incumbent may be exclusion from future benefits, a force which I do not model here because my model is static. Myerson (2008) highlights the commitment problem that an autocrat faces in

[3] The reasons why democracy gives rise to inefficiency are somewhat different in the two papers, however. I will return to this issue in Section 6.

promising benefits; I have simply assumed away this commitment problem. Guriev and Treisman (2020) develop a theory where the incumbent autocrat survives if the media say good things about her, and so an autocrat will invest resources in state-controlled media. Bidner et al. (2015) focus on "minimal democracies" where incumbents step down after they lose elections, and they ask why incumbents do so even if they have the power to resist the transition. Acemoglu et al. (2008, 2010, 2012, 2015) study relatively unstructured environments where institutions are minimal, and derive the features of "stable" regime types. In contrast, in most of the paper I take the regime structure (η, k, λ) as an exogenous parameter in order to focus on policy determination.

Finally, from a purely technical perspective, when $k = 0$ the right hand side in (5) is homeomorphic to the payoff in the "lottery Colonel Blotto" game studied by Friedman (1958), Snyder (1989), and Kovenock and Rojo Arjona (2019). None of these paper derive this functional form from a global game, as I do; rather, they assume it. In this sense, the present paper may be viewed as a "micro-foundation" of the reduced-form models in this literature. This literature is not concerned with public good provision.

2 Model

Society is a mass one of identical citizens indexed by $i \in [0, 1]$. Two politicians, an incumbent and a challenger, both seek power. The politicians simultaneously make promises to citizens. Based on these promises, citizens simultaneously choose either $a_i = 0$ ("support incumbent") or $a_i = 1$ ("support challenger"). If enough citizens support the challenger, the incumbent is replaced by the challenger. The game is as follows.

Stage 1: incumbent and challenger make promises. The incumbent (she) promises $\alpha_i \geq 0$ to citizen i if the citizen supports her, and $\lambda\alpha_i$ otherwise. This promise is only kept if the incumbent survives. Implicit in this setup is that a_i is observable to the politician. The parameter $\lambda \in [0, 1)$ represents the degree to which the regime is classically liberal, by which I mean that citizen i's right to enjoy α_i is protected independent of citizen i's own political activity. The incumbent's promises must satisfy the budget constraint:

$$\int_0^1 \alpha_i \, di = B_1 > 0. \tag{1}$$

The challenger (he) simultaneously promises $\omega_i \geq 0$ to individual i if the citizen supports him, and $\lambda\omega_i$ otherwise. This promise is kept only if the incumbent is overthrown.

The challenger's promises must satisfy the budget constraint:

$$\int_0^1 \omega_i \, di = B_2 > 0. \tag{2}$$

The numbers B_1 and B_2 are interpreted as total amount of tax revenue that each politician is able to raise if in office. It is natural to assume that $B_1 = B_2$, meaning that no politician enjoys an advantage, but that is not necessary for the analysis. Later, this policy space will later be expanded beyond simple redistribution to include a public good.

Citizen i's payoff is as follows:

	Incumbent replaced	Incumbent survives
Citizen i supports challenger	$\omega_i - k$	$\lambda\alpha_i - k$
Citizen i supports incumbent	$\lambda\omega_i$	α_i

(3)

Stage 2: citizens take collective action Every citizen i contemplates the vectors of promises $\boldsymbol{\alpha} = \{\alpha_i\}$ and $\boldsymbol{\omega} = \{\omega_i\}$, and then all citizens simultaneously choose $a_i \in \{0, 1\}$, with $a_i = 1$ meaning "support challenger." The cost of supporting the challenger is $k \geq 0$.

Stage 3: outcome The incumbent is replaced, if:

$$a = \int_H a_i \, di \geq 1 - \theta, \tag{4}$$

where H is subset of $[0, 1]$ with measure $\eta \in (0, 1]$. Citizens $i \in H$ are said to have political agency: if more than $1 - \theta$ of them choose $a_i = 1$, the incumbent falls. Citizens $i \notin H$ are politically powerless. The number a represents the political support for the challenger. The variable θ represents the incumbent's vulnerability (increasing in θ). Following Sakovics and Steiner (2012), θ is a draw from a random variable that is uniformly distributed over an interval $\left[\underline{\theta}, \overline{\theta}\right]$ that is a strict superset of $[0, 1]$.

Citizens' information. Citizen i is endowed with a private signal about θ, $z_i = \theta + \sigma\varepsilon_i$, where ε_i is i.i.d. independent of θ and has support $[-1/2, 1/2]$, and $\sigma \in (0, 1]$ is a scaling factor that determines the precision of i's signal.

2.1 Discussion of modeling assumptions

The index i could refer to a citizen or to an identifiable group of citizens. For example, i could represent "factory workers" or "workers in a given factory" or "the manager in a given factory." The important thing is that this individual or group has agency to support or oppose a politician, and that rewards can be targeted accordingly. For expositional brevity I will henceforth refer to i as a citizen.[4]

The action a_i represents citizen i's contribution to keeping the incumbent in office, or to removing her. The nature of the action will vary depending on the regime type. In authoritarian regimes, $a_i = 1$ represents taking a stand against the regime, including by protesting. In democracy a_i could represent voting; but since voting is typically secret, λ must be allowed to equal 1, which I will allow later in Section 6. A democracy with $\lambda < 1$ is also possible, and in this case a_i (which must be observable because $\lambda < 1$) represents observable forms of political support by individuals or groups, including get-out-the-vote efforts, the provision of favors and/or hiring, and monetary contributions.

For any constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, citizens either have a dominant strategy to support the challenger (if $(1-\lambda)\,\omega_i \leq k$), or they are engaged in a *coordination game*. To see this refer to matrix (3): any citizen who receives a promise $\omega_i > k/(1-\lambda)$ has conflicting incentives. The incentives to support the challenger are negative if the incumbent survives, and positive otherwise. Belief in the incumbent's survival, in turn, depends on the citizens' coordination in equilibrium. Therefore citizens are engaged in a coordination game.

The support $\left[\underline{\theta}, \overline{\theta}\right]$ of θ is strategically inconsequential: changing it will not change the politicians' equilibrium promises, although it will change the ex ante probability that the incumbent is replaced.

The parameter η represents the fraction of citizens who, collectively, can make the incumbent fall. This set becomes observable empirically when the incumbent falls. For example, in the 1917 Russian revolution these citizens were: the intelligentsia, the soldiers, and the factory workers – but not the farmers, arguably. (According to my model, this same set of citizens may well have passed up the opportunity to coordinate on replacing the incumbent many times before, due to low previous realizations of θ.)[5] [6] When

[4]Modeling citizens as a continuum allows me to use the law of large numbers as done in Myerson (1993).

[5]This modeling feature is historically accurate: indications that these citizens could coordinate and overthrow the incumbent had existed long before 1917.

[6]Low values of θ represent an incumbent's capacity to reduce or prevent coordination. For example, a capable chief of police or interior minister may be represented by a low θ.

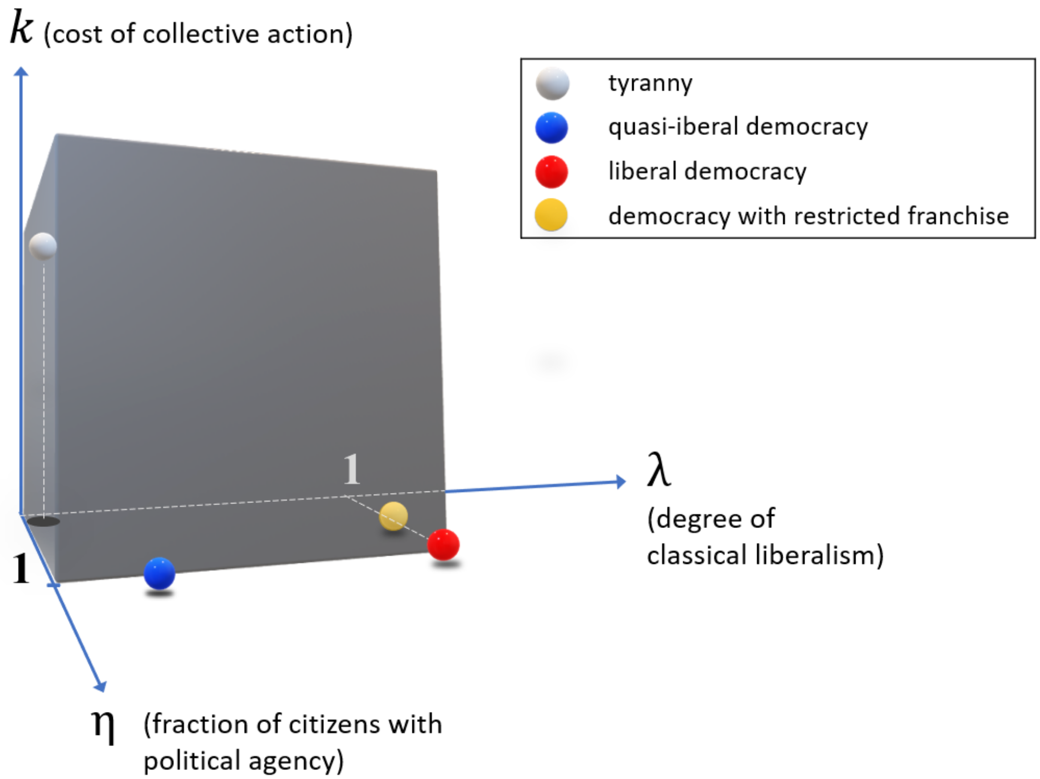

Figure 2: Regime space. Democracies lie at the base of the regime space.

incumbents are toppled by bloodless military coups, η represents the generals and perhaps the economic elites, but not broad strata of society. In democracy, η represents the voters (which may historically have excluded the poor, women, and any slaves).

The parameter k represents the additional cost of supporting the opposition instead of the incumbent. In non-democratic regimes k is large: it represents the individual cost of opposing the status quo (loss of job, beatings, imprisonment, or worse). In a democracy I assume $k = 0$ because whatever political action citizens engage in, be it voting or doing favors or extending patronage, the cost is the same whether it is exerted in favor or against the incumbent.

The parameter λ represents the fraction of a citizen's economic status that she is allowed to retain after supporting the politician that lost the struggle for power. A highly non-liberal system – one where the penalty for having supported the "wrong" politician is large – is captured by a λ close to zero. Conversely, a value of λ close to 1 represents a higly liberal system where one's political activity has almost no effect on one's economic status. The cost attached to λ is different from k in that: λ is not paid by citizens who support the challenger, if the revolt is successful; and λ is paid by citizens who support the incumbent, if the revolt is successful. For example, when an autocrat exiles political opponents after surviving a power struggle, the cost to the exiled is captured by λ rather than k because they would not have been exiled, had the struggle been successful. We can expect λ to be large in most democracies, but not necessarily to equal 1. I call a democracy "liberal" when $\lambda = 1$; this limit case is analyzed separately in Section 6 because the game is no longer a coordination game (refer to matrix (3)). I call a democracy with $\lambda < 1$ "quasi-liberal;" the interpretation of quasi-liberal democracy is discussed further in Section 9.1.

The triple (η, k, λ) represents, in effect, a set of "rules of the game" under which incumbent and challenger compete, and that they take as given. I call this set of rules a regime type. Figure 2 depicts the regime type space.

Definition 1 *The triple* (η, k, λ) *is a regime type.*

3 Citizens' equilibrium behavior

For expositional convenience, in this section only I restrict attention to the case $(\eta, \lambda) = (1, 0)$.

Given a constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, citizens are engaged in a "global game" similar to Sakovics and Steiner (2012), but with a technical wrinkle. Sakovics and Steiner (2012, Proposition 1) show that, in equilibrium, individual i supports the challenger if and only if $z_i \geq z_i^*$. As $\sigma \to 0$, all the thresholds z_i^* converge to a common limit:

$$\theta^* = \sum_g \frac{\alpha_g + k}{\omega_g + \alpha_g}.$$

This formula must be amended in our setting because we have a continuum of targetable units, so an integral sign must replace the summation sign. More substantively, in this setting it is possible that $\omega_i < k$. If that is the case then individual j supports the incumbent for sure (dominant strategy argument), and this violates a maintained assumption in Sakovics and Steiner (2012). In order to accommodate the case $\omega_i < k$, I need to extend their analysis with the following lemma.

Lemma 1 *Suppose* $(\eta, \lambda) = (1, 0)$. *Given a constellation of promises* $\{\boldsymbol{\alpha}, \boldsymbol{\omega}\}$, *the equilib-*

rium condition for incumbent survival is:

$$1 - \theta > \underbrace{\int_0^1 \frac{\omega_i - k}{\omega_i + \alpha_i} \cdot \mathbf{1}\left[\omega_i \geq k\right] \ di}_{incumbent\ vulnerability\ index} \ . \tag{5}$$

Proof. See the appendix. ■

The right hand side in (5) is an index of incumbent vulnerability given a constellation of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$. The integrand is between zero and one and thus so, too, is the index; this implies that, regardless of promises $(\boldsymbol{\alpha}, \boldsymbol{\omega})$, the incumbent survives when $\theta < 0$ and falls when $\theta > 1$. The mass of citizens who are promised $\omega_i \leq k$ do not contribute to the index, regardless of α_i: this reflects the fact that, in this case, supporting the challenger is a dominated (at least weakly) strategy. In the region $\omega_i \geq k$, the integrand is increasing in ω_i and decreasing in α_i.[7] These properties are intuitive: the incumbent is less vulnerable when the incumbent's promises are more generous and the challenger's promises are less generous. As expected, the index is nonincreasing in k, meaning that incumbent replacement is less likely when the cost of supporting the challenger is high. Condition (5) reduces to condition (5) in Sakovics-Steiner (2012) when $\omega_i \geq k$ for all i.

4 Politicians' equilibrium promises

This section shows that in any regime type (η, k, λ) the incumbent will treat citizens who have political agency equally, but the challenger may not. That the incumbent always chooses equal treatment is not obvious; the intuition for this result will be developed later in this section – and further strengthened in Section 6. The fact that the challenger may deviate from equal treatment is due to the disadvantage embodied in k: if the challenger treats everyone equally he risks spreading his resources too thin. I will provide more intuition for the challenger's equilibrium strategy after stating this section's result.

A politician's strategy is a probability distribution from which promises are independently drawn. Distributions can be citizen-specific, for example: citizen i is promised 2 and citizen i' is promised 4 with probability 1/2, and 6 with probability 1/2. I will restrict attention to the class of equilibria in which strategies do not condition on the identity of identical citizens.

Definition 2 *A strategy is called symmetric if promises to every citizen with political*

[7] The integrand goes from 0 when $\omega_i = k$, to 1 when $\omega_i = \infty$; and from $\left(1 - \frac{k}{\omega_i}\right) \in [0, 1]$ when $\alpha_i = 0$, to 0 when $\alpha_i = \infty$.

agency are drawn from a single probability distribution, and promises to every citizen without political agency are drawn from another single probability distribution.

Symmetric strategies are natural in this setting because strategies that condition on a citizen's identity make it easier for the opponent to invest resources on the most responsive citizens and "save" on the rest. The next proposition shows that there is a unique equilibrium in symmetric strategies, and characterizes it.

Proposition 1 (egalitarian vs inequitable equilibrium promises) *For candidate* $j = 1, 2$ *denote:*

$$\overline{B}_j = \frac{(1-\lambda)}{\eta} B_j,$$

and let:

$$h(\alpha; k) = k + \sqrt{k\alpha + k^2}. \tag{6}$$

There is a unique equilibrium in symmetric strategies, and it has the following features.

1. *Citizens without political agency are promised zero.*
2. *The incumbent promises citizens with political agency an egalitarian distribution, i.e.,* $\alpha_i^* = B_1/\eta$ *for all* $i \in H$.
3. *If* $\overline{B}_2 \geq h\left(\overline{B}_1; k\right)$ *the challenger promises citizens with political agency an egalitarian distribution, i.e.,* $\omega_i^* = B_2/\eta$ *for all* $i \in H$. *This is the case for* k *small enough.*
4. *If* $\overline{B}_2 < h\left(\overline{B}_1; k\right)$ *the challenger promises citizens with political agency an inequitable distribution: some of them, chosen at random, are offered* $h\left(\overline{B}_1; k\right)/(1-\lambda)$, *the rest are offered zero.*

The budgets $\overline{B}_j = (1-\lambda) B_j/\eta$ represent "rescaled budgets."[8] The rescaled budgets $\overline{B}_1, \overline{B}_2$ are more generous when η and λ are small. This is intuitive: when η is small more (per capita) is left to distribute to citizens with political agency, after the other citizens are expropriated; and when λ is small the incentives available to the politicians are, in effect, more powerful relative to k.

Part 1 is obvious: no rational politician would make any positive promises to citizens who have no political power. I now provide some in-depth intuition for parts 2 and 3,

[8] This rescaling is used in the proof of Proposition 1 to apply Lemma 1. The proof shows that any game with $(\eta, \lambda) \neq (1, 0)$ and budgets B_1, B_2 is strategically equivalent to a game with $(\eta, \lambda) = (1, 0)$ and budgets $\overline{B}_1, \overline{B}_2$.

that is, for why the incumbent will treat citizens with political agency equally, but the challenger may not. To fix ideas, suppose $(\eta, \lambda) = (1, 0)$, that is, all citizens have political agency and illiberality is maximal. Then the incumbent seeks to *minimize* the right hand side in (5) subject to the budget constraint (1). The right hand side in (5) may be written as:

$$\int v(\alpha_i, \omega_i; k) \; di, \tag{7}$$

where:

$$v(\alpha, \omega; k) = \left(\frac{\omega - k}{\omega + \alpha}\right) \cdot \mathbf{1}[\omega \geq k]. \tag{8}$$

Since v is a convex function of α, if the challenger treats all citizens symmetrically then (7) is minimized by promising every citizen an equal share of the budget. The challenger's problem is somewhat more complex: he seeks to *maximize* (7), but v is *not* a globally concave function of ω (Figure 3 plots v as a function of ω). Therefore the challenger may choose not to treats all citizens equally. To understand why, it helps to form the *concave envelope* $\overline{v}(\alpha, \omega)$. Figure 3 plots v and $\overline{v}$ as a function of ω: the key features are that $\overline{v}$ lies above v, and that the challenger can in fact achieve the value $\overline{v}$ by the following inequitable strategy. Suppose the incumbent promises α to a positive mass of voters. If the challenger's available resources b_2 for that group fall below $h(\alpha; k)$, then the challenger's best response is to promise exactly $h(\alpha; k)$ to each citizen with political agency with some probability, and zero with complementary probability, such that the budget constraint is met by the law of large numbers. Though this strategy is *inequitable even among citizens with political agency*, it achieves the optimal value of $\overline{v}(\alpha, b_2; k)$, which exceeds the equal-treatment value $v(\alpha, b_2; k)$.[9] We see from expression (6) that $h(\alpha; k) \to 0$ as $k \to 0$. Thus for small k the region where v is non-concave is small and then it is optimal for the challenger to treat citizens equally for most values of b_2. Conversely, for large k the region of non-concavity is large, and inequitable treatment is a best response for many values of b_2. A large k represents an incumbent advantage so large that if the challenger were to spread his resources equally among the citizens with political agency, he would be spread too thin.

5 Provision of an egalitarian public good

In this section I enlarge the policy space by adding a policy which I call an *egalitarian public good*. Set $B_1 = B_2 = B$, which means that no politician enjoys an advantage. I

[9] The challenger picks the lucky citizens randomly to avoid them being "picked off" by the incumbent.

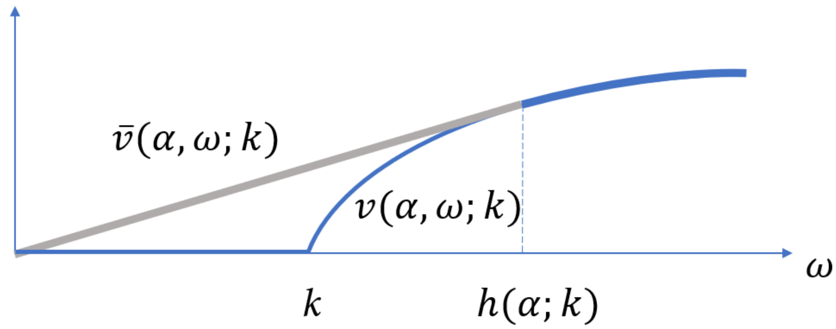

Figure 3: Why the challenger may redistribute inequitably. The incumbent promises α. If the challenger's budget is below $h(\alpha; k)$, the challenger benefits from making inequitable promises: in fact, the incumbent's vulnerability is maximized by promising $\omega = h(\alpha; k)$ with some probability, and $\omega = 0$ with complementary probability. If the challenger's budget is greater than $h(\alpha; k)$, the challenger benefits from promising the same benefit to every citizen with political agency.

assume that either politician can offer a public good that gives exactly $G > 0$ to each citizen; or, alternatively, the politician can freely redistribute B.

I interpret the egalitarian public good as the use of available state capacity (i.e., the tax revenue B) to pay for a policy with broad-based benefits. For example: using coercive state powers to procure grain (B) that is then exported to pay for agricultural machines that improve the productivity of collective farms (G), or for universal education/health care (G), or for national defense (G). Just like the promises in the previous section, so too the benefits of the public good are denied if a citizen failed to back the winning politician; this implies that the public good is *excludable*.[10]

Definition 3 (socially efficient vs agent-optimal policies) *Set $B_1 = B_2 = B$. The socially efficient policy is to provide the public good if and only if $G \geq B$. The agent-optimal policy is to provide the public good if $G \geq B/\eta$, else to redistribute all the budget to the citizens with political agency.*

This definition of agent-optimality looks at outcomes from the perspective of a citizen who has political agency. It compares the value of the public good with the value of redistribution *after citizens without political agency have been expropriated*.[11] The agent-optimal policy inefficiently under-provides the public good relative to social efficiency

[10] In authoritarian regimes, citizens can be excluded from the enjoyment of most public goods through coercion, incarceration, or worse.

[11] This definition abstracts from distributional considerations (inequality). Inequality is addressed separately as an equilibrium outcome throughout this paper.

whenever $\eta < 1$ because citizens with political agency do not internalize the entire population's benefits of consuming the public good (or, equivalently, they overrate the benefits of redistribution). Since the benefits of the public good cannot be targeted among the citizens with political agency, no politician will want to promise the public good if $G < B/\eta$: therefore public good *overprovision* relative to the agent-optimal policy does not arise in equilibrium. However, *underprovision* – failing to provide the public good when $G > B/\eta$ – may arise.

Proposition 2 ***(provision of an egalitarian public good)*** *Suppose $B_1 = B_2 = B$, and denote:*

$$\begin{aligned}
\overline{B} &= \frac{(1-\lambda)}{\eta} B \\
\overline{G} &= (1-\lambda) G \\
M &= \max\left[\overline{B}, \overline{G}\right]
\end{aligned}$$

There is a unique equilibrium in symmetric strategies, and it has the following features.

1. *Citizens without political agency are promised zero whenever redistribution is promised.*

2. *The incumbent promises the agent-optimal policy and equitable treatment among the citizens with political agency.*

3. *The challenger promises the agent-optimal policy and equitable treatment among the citizens with political agency if and only if:*

$$v(M, M; k) \geq \overline{v}\left(M, \overline{B}; k\right), \tag{9}$$

   else, the challenger will promise unequal redistribution among the citizens with political agency. Given any pair $\left(M, \overline{B}\right)$ condition (19) holds for any k that is small enough.

4. *For any given value of B, parameters $G > B/\eta$ and $k > 0$ exist such that the challenger does not promise the public good even though it is agent-optimal.*

5. *For both challenger and incumbent, the set of values (B, G) such that the public good is promised grows with η.*

6. *The probability that the incumbent promises the public good is independent of λ. The challenger promises inequitable redistribution for all $\lambda > (G-k)/G$.*

Proof. See the Appendix. ■

Part 2 says that the incumbent's promises are agent-optimal. When redistribution is the efficient policy, all the citizens with political agency are promised the same amount. In contrast, the challenger does not necessarily promise the efficient policy. Part 3 says that if condition (9) fails, which happens if k is large, the challenger will choose inequitable redistribution. Part 4 says that inequitable redistribution will sometimes be promised rather than an efficient public good, and the proof of the part 4 indicates that this happens when k is large enough and the public good is not much more efficient than redistribution. This inefficiency arises because promising the public good requires equal treatment; when k is large, equal treatment is strategically costly for the challenger because it spreads his resources too thin. This is the same force toward unequal treatment as in Proposition 1. Part 5 says that the public good is provided more often as η increases. This because by promising the public good, politicians are forced to "waste incentives" on citizens who do not have political agency. As η grows the set of citizens with political agency increasingly overlaps with the entire population, and so the "wasted incentive" effect progressively disappears for both politicians. Part 6 is subtle. As the system becomes more classically liberal, the incumbent's promises don't change, they remain agent-optimal. But, when the system is sufficiently liberal, the challenger's cannot avoid promising redistribution, even when redistribution is inefficient; indeed, note that the condition $\lambda > (G - k)/G$ does not involve B, and therefore is orthogonal to the (in)efficiency of redistribution. This effect arises because in a very classically liberal system a politician's promise carries little incentive power, and so the challenger's disadvantage due to k looms large in comparison. In this scenario, as discussed before Proposition 1, if the challenger spread his resources equally he would be spread too thin.

I have assumed at the beginning of this section that the public good is excludable. If the public good is non-excludable, Proposition 2 does not hold and the public good will not be provided in equilibrium.

Remark 4 (non-excludable public goods) *A non-excludable public good will not be promised in a non-liberal regime, because it generates no incentive to support the politician who promised it. This is because the politician cannot "take back" non-excludable public goods from citizens who failed support him/her.*

Of course, in the context of authoritarian regimes where citizens can be excluded from the enjoyment of most public goods through coercion, incarceration, or worse, practically all public goods may be regarded as excludable.

6 Liberal and quasi-liberal democracy

A democracy with full franchise is the special case where $(\eta, k) = (1, 0)$. Because $k = 0$ the game is symmetric: the incumbent enjoys no strategic advantage over the challenger.[12] The incumbent is replaced according to condition (4).[13] In this section I distinguish between two types of democracy: an quasi-liberal one ($\lambda < 1$), that will generate efficient provision of the egalitarian public good; and a liberal one ($\lambda = 1$) that will not.

The notion of "quasi-liberal democracy" is novel, and certain of its features requires intepretation. I defer the interpretation to Section 9.1. The "liberal democracy" scenario $(\eta, k, \lambda) = (1, 0, 1)$ coincides with the voting game studied in most theoretical papers. Since $\lambda = 1$ promises are *not* conditional on a_i, so a_i may properly be interpreted as anonymous voting. Mathematically, this case is degenerate because the citizen's incentive to vote vanishes (to check this, plug $k = 0, \lambda = 1$ into matrix (3)). I resolve this ambiguity with a standard assumption: I assume that citizen i votes for the candidate who promises the most. This assumption, which is standard in voting games, turns the coordination game into a dominant strategy game where voters do not seek to coordinate but, rather, vote without regard to each other's vote. The takeaway is that the calculus of voting in the "liberal democracy" scenario is qualitatively different from that in all other nearby parameter values. This difference results in different equilibrium policies, as shown in the next result.

Proposition 3 (provision of egalitarian public good in democracy) *Fix B, G, and set $(\eta, k) = (1, 0)$ (i.e., costless voting and full franchise).*

1. ***(quasi-liberal democracy: efficient provision)*** *Suppose $\lambda < 1$. Then both politicians promises the socially efficient policy and equal treatment across the entire population.*

2. ***(liberal democracy: inefficient provision)*** *Suppose $\lambda = 1$. Then neither politician promises the public good if $G \leq B$, and this is efficient. If $G \in (B, 2B)$ both politicians promise the public good with probability $(G - B)/B$, and this is inefficient underprovision. If $G \geq 2B$ both politicians promise the public good, and*

[12] However, payoff *levels* need not be the same: one or the other politician may enjoy a *non-strategic* advantage, meaning that the prior distribution of θ may favor either.

[13] This assumption means that replacement occurs in proportion to the challenger's vote share, and not when his vote share exceeds $1/2$. This assumption is not uncommon in the voting literature. It is made here to avoid changing the game structure discontinuously in the neighborhood of $(\eta, k) = (1, 0)$. In Remark 6 I extend the analysis to the more natural case where replacement occurs when the challenger's vote share exceeds $1/2$.

this is efficient. When the public good is not promised, inequitable *redistribution is promised.*

Proof. Part 1. Follows from Proposition 2 parts 2 and 3, since $k = 0$ and $\eta = 1$ here.

Part 2. When $\lambda = 1$ I assume that citizen i chooses $a_i = 1$ if and only if $\alpha_i \leq \omega_i$. Then the condition for incumbent replacement (4) rewrites as:

$$\int_0^1 \mathbf{1}(\alpha_i \leq \omega_i)\ di \geq 1 - \theta. \tag{10}$$

This means that the challenger replaces the incumbent if the challenger's vote share exceeds $(1 - \theta)$. Since θ is distributed uniformly, maximizing (or minimizing) the probability of event (10) is the same as maximizing (or minimizing) the vote share. Therefore this game between politicians is exactly equal to the "proportional system" analyzed in Lizzeri and Persico (2001). The result for our case is found in their Theorem 3, which studies a case where politicians maximize their vote share. ■

The nature of political competition is qualitatively different between liberal ($\lambda = 1$) and quasi-liberal ($\lambda < 1$) democracy. In a liberal democracy politicians target individual voter preferences. This is because citizen i votes based on whether politician 1's promise *to herself* is greater than politician 2's. In a quasi-liberal democracy, instead, politicians target equation (5), which is monotonically related to the *common belief* of the likelihood of regime change. In fact, in any non-liberal system, citizen i contemplates the *entire distribution* of promises from both candidates, uses these distributions to compute a *common belief* of regime change, and then acts accordingly. The next example illustrates how the two modes of political competition result in different equilibrium policies.

Example 1 (drivers of egalitarian public good underprovision in liberal democracy) *Suppose $B = 1$ and $G = 1.5$, so that G is efficient. If both candidates were to promise G for sure, then each of their vote shares would be equal to 1/2. In a liberal democracy, candidate 2 could deviate and promise* $1.5 + \varepsilon$ *to almost* $2/3$ *of the citizens, and zero to the rest. This deviation delivers a vote share of almost 2/3, which is better than 1/2. So "G for sure" is not an equilibrium in a liberal democracy. Candidate 2's deviation is inefficient because it makes 2/3 of the voters vanishingly happier than* G, *and 1/3 much less happy: but the deviation pays off because the intensity of the voters' preferences does not matter. This scenario is illustrated in Figure 4. In a quasi-liberal democracy, the same deviation would deliver candidate 2* less than $1/2$ *when plugged expression (5), due to the convexity of the expression in* $\boldsymbol{\omega}$.

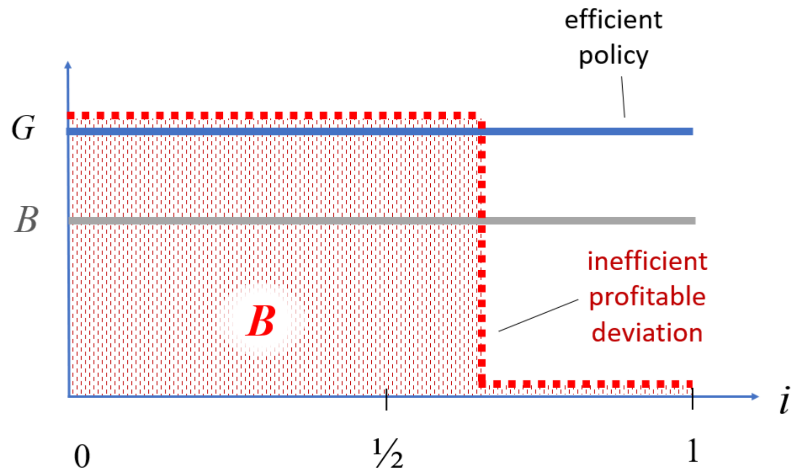

Figure 4: Incentive to underprovide an egalitarian public good in a liberal democracy. The efficient policy is G, but it is not an equilibrium for both politicians to promise G. If a politician promises G her opponent benefits from deviating to redistribution. The deviation is carefully targeted to win as many votes as possible, which is more than 1/2 of the votes, as illustrated by the dashed red line. The area under the dashed red line equals B as required to meet the budget constraint.

Example 1 illustrates why public good provision is more efficient in a quasi-liberal than in a liberal democracy. This is *not*, as in McGuire and Olson (1996), because in liberal democracy politicians only care about 50% of the votes: indeed, in this section politicians maximize the *vote share* (refer back to the discussion in Footnote 13). Thus, the offending deviation in Figure 4 maximizes the *vote share* – and yet it is inefficient. The reason that maximizing the vote share does not coincide with maximizing welfare is that the intensity of voter preferences does not feature in the vote share. Indeed, the politicians' objective function (10) only depends on whether $\alpha_i \leq \omega_i$, but not on the magnitudes of α_i and ω_i In contrast, in a quasi-liberal democracy the intensity of voter preferences features in the politicians' objective function, because expression (5) depends on the magnitudes of α_i and ω_i. This being understood, it makes sense that politicians promise more efficient policies when their incentives take into account preference intensity.

The next theorem is the first main result: it characterizes the entire regime type space according to the efficiency of public good provision.

Theorem 5 (efficiency of egalitarian public good provision across regime types)

1. ***(non-democracies: inefficient provision)*** *Any non-democratic system, i.e., one where* $(\eta, k) \neq (1, 0)$ *, fails to achieve efficient provision for some pair* (B, G) *.*

2. ***(quasi-liberal democracy: efficient provision)*** *Quasi-liberal democracy, i.e.,* $(\eta, k, \lambda) = (1, 0, < 1)$ *achieves socially efficient provision for any pair* (B, G)*.*

3. ***(liberal democracy: inefficient provision)*** *Liberal democracy, i.e.,* $(\eta, k, \lambda) = (1, 0, 1)$ *fails to achieve efficient provision for some pair* (B, G) *.*

Proof. Efficient provision requires both politicians to promise the efficient policy, because in equilibrium both politicians obtain power with positive probability.

Part 1. Inefficient provision when $k > 0$ is established in Proposition 2 part 4. Inefficient provision when $\eta < 1$ follows from Proposition 2 parts 2 and 3, which establish agent-optimal (but not necessarily socially efficient) provision for any triple (B, G, η) for k small enough. It is then a matter of choosing the triple (B, G, η) such that $B < G < B/\eta$, so that public good provision is socially efficient but not agent-optimal.

Part 2. The efficiency of quasi-liberal democracy for any (B, G) is stated in Proposition 3 part 1,

Part 3. Proposition 3 part 2 states that liberal democracy is inefficient for some (B, G) . ■

The reason why quasi-liberal democracy performs better than all other systems is the following. Any system with $\eta < 1$ suffers from an under-representation problem, which obviously leads to underprovision of the public good. But even if $\eta = 1$, a non-democratic system ($k > 0$) suffers from the problem that the challenger will sometimes "go for broke" and focus his promises on a subset of the citizens to cope with the strategic disadvantage created by $k > 0$. In order to do so, the challenger has to give up on the public good even when it is efficient. Finally, the problem with liberal democracy is that the voter's preference intensity is irrelevant to the outcome and so, as illustrated in Example 1, politicians have an incentive to treat some citizen as well as needed, and fully expropriate the rest.

I close this section by generalizing the condition that triggers incumbent replacement. So far, I have restricted attention to condition (4), which means that the challenger wins if his vote share $a \geq 1 - \theta$. I have done this for comparability: by keeping the incumbent replacement rule fixed as the parameters (η, k, λ) vary, I was able pinpoint the source of the difference in performance across regime types. However, in a democracy a majoritarian condition may be more natural. I address this in the next remark.

Remark 6 (extension to majoritarian rule of incumbent replacement) *In a democracy the condition that triggers incumbent replacement is often* $a \geq 1/2$*, rather than condition (4) as currently assumed. In Appendix A I show that the results in Proposition*

3 part 1 extend verbatim to a transition rule arbitrarily close to $a \geq 1/2$. The results in part 2 also extend with minor changes to the case $a \geq 1/2$ (cf. Theorem 2 of Lizzeri and Persico 2001, where candidates maximize the probability of winning the election rather than their vote share.) Theorem 5 continues to hold verbatim if the replacement rule is $a \geq 1/2$.

7 The virtue of liberalism: beyond egalitarianism

Suppose each candidate can either offer redistribution or an excludable non-egalitarian public good $\widetilde{G} \sim U[0, \overline{g}]$ that gives g_i to citizen i with uniform probability.

I interpret the non-egalitarian public good as the use of available state capacity (i.e., the tax revenue B) to pay for a policy with unequally-distributed benefits. For example, using tax revenue to support institutions that enable the appropriation of rents by innovators and capitalists, at the expense of workers. These institutions include strong capital and IP protection, functioning courts, and modest labor protections. This type of policy may well be efficient but it creates inequality.

Theorem 7 (efficiency of non-egalitarian public good provision across systems)

1. *The non-egalitarian public good $\widetilde{G}$ is not promised by either politician if it is not agent-optimal, i.e., if $\mathbb{E}\left(\widetilde{G}\right) < B/\eta$.*

2. ***(non-liberal system: parameter region of inefficient provision)** Assume $\lambda < 1$. If $\mathbb{E}\left(\widetilde{G}\right) = c\,(B/\eta)$ and $c \in (1, 1.25)$ the non-egalitarian public good is agent-optimal but, for k small enough, it is not an equilibrium for both politicians to provide it. In this parameter region both politicians promise egalitarian redistribution among the citizens with political agency.*

3. ***(liberal democracy: efficient provision)** Assume $(\eta, k, \lambda) = (1, 0, 1)$. Then the non-egalitarian public good is provided if and only if it is agent-optimal, i.e., if $\mathbb{E}\left(\widetilde{G}\right) > B/\eta$.*

Proof. See the Appendix. ■

Part 1 is obvious: there is no point in providing a policy that has lower mean *and* less flexibility than redistribution. Part 2 is intuitive: we know from Proposition 1 that when k is small enough egalitarian redistribution among the citizens with political agency is a best response to itself; therefore, a policy $\widetilde{G}$ that is very unequal cannot be a best response to redistribution unless it is much more efficient than redistribution; the proposition indicates that $\widetilde{G}$ must be more than 25% more efficient than redistribution in order to be appealing.

While Part 2 restricts attention to the case where k is small, inefficient underprovision is not limited to this case: intuitively, this is because the incumbent always prefers egalitarianism among the citizens with political agency, and so the incumbent would prefer to avoid the distributional inequality that comes with promising $\widetilde{G}$.

Corollary 1 (most socially efficient system) *Consider any triple* $\left(B, G, \widetilde{G}\right)$*. An egalitarian public good with social value* G *is efficiently provided by a quasi-liberal democracy. A non-egalitarian public good with social value* $\mathbb{E}\left(\widetilde{G}\right)$ *is efficiently provided by a liberal democracy.*

Proof. Follows from Theorems 5 and 7. ■

8 Regime transition

A complete theory of regime transition is beyond the scope of this paper. In this section I limit myself to some observations which can hopefully help shape a future theory of regime transition.

The first observation is that a regime type in the sense of Definition 1 is a non-excludable public good because it cannot be personally tailored to a citizen. Therefore, by Remark 4, if $\lambda < 1$ voters cannot be incentivized to support a politician through the promise of a favorable regime type. This is a remarkable observation. The intuition is that in my model citizens are only incentivized by benefits that *can be revoked* for individual failure to act. Since a regime type is a non-excludable public good, a promise of regime change carries no incentive power whenever the current regime features $\lambda < 1$.

The above observation suggests the next one: transition from a regime with $\lambda < 1$, to the extent that it happens, does not come about as the *fulfillment of a promise* by the winner of the political struggle. Instead, I posit that it comes about *in anticipation of a coming political struggle*, through bargaining between the current incumbent and the citizens with political agency. In this view, the incumbent and the citizens with political agency bargain to shape the rules that will govern the upcoming contest for power between incumbent and challenger.

The third observation is that, whatever form this bargaining takes, it will be shaped by the conflict of interest between the bargaining parties. What are these conflicts of interest?

One is between incumbent and citizens with political agency regarding the contestability of the regime. All else equal, the incumbent's bliss point is $k = \infty$ (no risk of replacement), but the citizens with political agency's bliss point is $k = 0$ (when $k = 0$ public good provision is always agent-optimal). So we can expect the citizens with political agency to push for a more contestable regime than the incumbent is willing to grant.

The fourth observation has to do with the nature of the public goods. Depending on whether the available public goods are egalitarian or not, different regime types are most suited for optimal provision from the perspective of citizens with political agency. Therefore, we expect the citizens with political agency to seek regime transition when the nature of the available public good changes. In Section 9.3 I interpret the availability of either type of public good as related to the distance from the technology frontier. If we accept that interpretation, it follows that the citizens with political agency will prefer a non-liberal regime (including a quasi-liberal democracy) in developmental states , and a liberal democracy in advanced economies, because each regime type efficiently delivers the type of public good that is available given the distance from the frontier. As a corollary, when an economy moves closer to the technological frontier, its citizens with political agency should bargain for a more liberal system. This seems to be the case in reality, because as countries get wealthier they tend to move toward more liberal systems.

The fifth observation has to do with voluntary expansion of political rights. As in Lizzeri and Persico (2004), here also the citizens with political agency may have a preference for voluntarily expanding the set H. The reason is that under certain parameter values the outcome of the political contest is not agent-optimal: the challenger fails to promise an agent-optimal public good (Proposition 2 part 4), and so the welfare of citizens with political agency is reduced if the challenger prevails. In these circumstances, the citizens with political agency benefit from expanding the set H because doing so increases the set of parameters under which the challenger promises the public good (Proposition 2 part 5).

9 Discussion and related literature

9.1 Realism of quasi-liberal democracy

I have called the case $(\eta, k, \lambda) = (1, 0, < 1)$ "quasi-liberal democracy" because, while the challenger is not disadvantaged and the franchise is full, the system is not perfectly liberal: promises to citizens or groups are *conditional* on their individual voting behavior.

Democracy with conditional promises has not been studied in the theoretical literature, but it is worth studying for several reasons. First, promises to groups (a union, a locality, an identity group etc.) conditional on the group's vote "coming through" are commonplace in democratic politics.[14] Second, voting is not the only thing: most other forms of political support – campaign contributions, get-out-the-vote efforts, and the provision of favors and patronage – are observable and rewards can be conditioned on them.[15] Third, in certain countries even the personal vote is said to be observable to some degree, and so benefits can be conditioned on the casting of individual votes.[16] The point is that conditional promises are realistic in many real-world democracies: in this paper's language, many democracies are somewhat non-liberal.

Citizens perceive quasi-liberal democracy as a coordination game and, in the limit equilibrium where $\sigma \to 0$, they unanimously support the winning candidate. Of course such uniform consensus is too stylized to be observed in actual democratic elections: in Appendix A I sketch out a more realistic variant of the model where *only a pivotal subset of voters* need to coordinate on the winning candidate, and the rest of the voters are free to vote "individualistically." Having said this, I view voter coordination in quasi-liberal democracy as a feature, not a bug, because it is reminiscent of the consensus that is the hallmark of corporatist/consensual/coordinated democracies: see Section 9.2.

9.2 Interpreting quasi-liberal democracy as as corporatism

The distinction between pluralist (such as the US) and corporatist/consensual/coordinated democracies (such as most European countries, including Germany and Scandinavia) has been much discussed in political science: see, e.g., Lijphart and Crepaz (1991), Siaroff

[14] For example, then-presidential candidate Gov. Perry from Texas said to Iowans in 2011: "If you'll have my back in Iowa on Jan. 3, I'll have your back in Washington, D.C., for four years." See Saenz (2011).

[15] In many democracies doing favors (in hiring, in bureaucratic actions, etc.) is a form of political activity if the beneficiary is a party official or the issue is politically sensitive. Favor-doers often experience career advancement while the beneficiary party is in power. In the US parties are weaker, and the conditionality of promises is also weaker. At the local level, however, the observable efforts of citizen activists (getting out the vote or protest organizing) may be rewarded by the local party machine or other local organizations. Finally, financial contributions are observable and may be rewarded with favorable policy decisions. For all these forms of political support there is a clear incentive to jump on the winner's bandwagon because the loser is generally not in a position to deliver the benefits; this is precisely the incentive to coordinate featured in this paper.

[16] See Schaffer (2004) for mentions of many clever ruses through which anonymity is broken. On the conditionality of future benefits refer to the discussion of "probabilistic selective incentives" in Brusco et al. (2004), e.g., on p. 84 they write that "voters will view the future flow of personalized handouts to them and their families as conditional on their supporting the clientelist party." On a long term relationship between an individual voter and a party, see Finan and Schechter (2012).

(1999), and Lijphart (2012). This literature classifies democracies according to the way in which disparate economic interests coalesce into industrial policy. By industrial policy I mean the set of laws and regulations that constrain the production and distribution of economic rents.

Two polar cases are identified: in corporatist/consensual/coordinated democracies political parties are strong and they deeply permeate economic life. In these democracies, industrial policy is hammered out in informal and opaque bargaining between political parties, bureaucrats, and the institutions representing capital and labor (trade associations, unions). This bargaining is not based on clear rules and minimal-winning coalitions but, rather, on compromises that reflect the intensity of preferences. The policy outcome is said to be relatively consensual, coherent, and durable. By contrast, in pluralist democracies (typically, the US) political parties are relatively weak and have relatively little influence in economic life. Policy arises out of a sequence of independent majoritarian games played by minimal-winning, short-lived, and ad-hoc coalitions of interests. Every one-shot contest in which policy is determined is said to have clear winners and losers, and the resulting industrial policy is thought to be piecemeal, and always up for revision.

The equilibria when $\lambda < 1$ and when $\lambda = 1$ reproduce the features of corporatist/consensual/coordinate and pluralist democracies, respectively. Indeed, $\lambda < 1$ means that political parties have a strong hold on the economy. Then the theory predicts that coalitions will be universalistic because voters coordinate perfectly, and that equilibrium policy tends toward egalitarianism (seen most clearly from Proposition 1 parts 2 and 3, after noting that $k = 0$). When $\lambda = 1$, instead, parties don't have much of a hold on the economy. Then the theory predicts that candidates will form opportunistic coalitions (seen most clearly in the coalition that defeats the public good G, described in Figure 4 and Example 1), platforms do not reflect the intensity of preferences, and policy tends toward non-egalitarianism (seen most clearly from the fact that a public good is underprovided if it is egalitarian, see Theorem 5 part 3 and the discussion following it). Therefore equilibrium performance when $\lambda > 1$ differs from that when $\lambda = 1$ in ways that, qualitatively, correspond to the corporatist/consensual/coordinated vs pluralist dualism. The advantage of my analysis is that the correlation is not a descriptive statement: it arises in equilibrium due to variation in λ.

9.3 Why liberal democracy fosters innovative growth

I interpret the public good as the growth that may be enabled by government policy. According to this interpretation, the role of government is to orient the available state capacity (captured by B, its ability to tax) toward the goal of delivering growth. The theory distinguishes between growth with broad benefits (as in Sections 5 and 6) and growth with concentrated benefits (as in Section 7). When a country is at the technological frontier, as is the case for the US, growth has to come from internal innovation and shrewd capital allocation. This type of growth requires government investment in the institutions that enable the (unequal) appropriation of rents by innovators and capitalists; these include strong IP protection, functioning courts, and weak labor protections. Generating this type of growth requires accepting inequality between innovative elites and the rest of the citizens. I interpret the non-egalitarian public good $\widetilde{G}$ from Section 7 as the outcome of this type of growth. By Corollary 1, liberal democracy is the best system for providing such growth. In contrast, quasi-liberal democracy penalizes innovative growth as defined here.

If we accept the argument in Section 9.2 that liberal democracy is closer to the US and quasi-liberal democracies are closer to consensual democracies of Europe, an intriguing prediction emerges: that US democracy is expected to foster more innovative and more unequal growth, and European democracies to foster less innovative but more-equitable growth. This prediction seems reasonably factual.

9.4 Theoretical predictions regarding democracy and development

A number of countries have developed quickly under non-democratic regimes: in the 20th century these include Chile, China, Russia, South Korea, Singapore, and Taiwan. A comparison with India, historically a slow-growing democracy, has led some to conclude that developing countries may achieve higher growth under authoritarian governments than under democracy, because removing democratic constraints allows leaders to be more effective.[17] Empirically, it is controversial whether democracy promotes or hinders growth.[18]

The present theory suggests that blanket statements like "democracy helps" or "democ-

[17] See Dick (1974) for this argument.

[18] Przeworski et al. (2000) find that the effect of democracy on growth is ambiguous, Barro (1996) finds that democracy reduces growth, and most recently Acemoglu et al. (2019) find the opposite.

racy hurts" growth are too coarse. Which system delivers the most growth depends on the *distributional features of the available growth opportunities*. If the available growth opportunities for a given country are egalitarian then a quasi-liberal democracy or even a non-democracy could perform better than a democracy. If, instead, the available growth is non-egalitarian, then democracy performs best.

I submit, tentatively, that catch-up growth opportunities are more likely to be egalitarian than the innovative type of growth discussed in Section 9.3. This is because catch-up growth relies on replicating existing foreign technology, obviating the need to reward innovators hugely. Even the rents to capital can be reduced by forcing state-sponsored capitalists to reinvest their profits into local projects rather than invest abroad or consume lavishly. Such equitable growth can be achieved through command-and-control industrial policy.

According to this argument, the political incentives generated by consensus democracies ($\lambda < 1$) are best for delivering catch-up growth. Intriguingly, this is precisely Eichengreen's (2008) explanation for why European democracies (all consensus democracies) were able to quickly catch up to the US after the second world war.[19]

9.5 Extractive autocracies

All non-democratic systems are bad according to Corollary 1: they are dominated by a democracy of some type, either liberal or quasi-liberal, in terms of efficiency of public good provision. Non-democracies fail to provide the public good efficiently for two reasons. First, if the set of citizens with political agency is restricted ($\eta < 1$) then public good provision may be at best agent-optimal, but never socially efficient due to the under-representation problem discussed after Theorem 5. Indeed, when the set of citizens with political agency is very narrow, that is, $\eta \ll 1$, even a very efficient public good will not be provided. Second, when the incumbent is replaced, a social cost $k > 0$ is paid. These narrow autocracies are purely extractive, and they truly have no redeeming features.

9.6 Observable implications

This section deals with observable implications of the theory. I have in mind a scenario where, at certain times in her tenure, the incumbent risks being replaced. (In the case of democracy this happens periodically). I consider a setting where the game of Section 2 is played repeatedly at $t = 1, 2, \ldots$, between the time-t incumbent and a time-t challenger. At

[19] The same argument holds for Japan.

every time t a certain B_t and G_t are available. I assume that the stage Nash equilibrium in Proposition 1 is played every time, thus abstracting from strategic links between periods.

In this scenario, the model gives a number of predictions. First, since I look at the limit where $\sigma \to 0$, all citizens almost always back the successful politician, and they incur the cost k if and only if the incumbent is replaced. If we are looking at a non-democratic system, this means that there are many *unobserved opportunities* to launch a coup or revolution, but the coup/revolution (and the cost k) *will only be observed if successful*.

Second, suppose the non-egalitarian public good is not feasible (perhaps because, as argued in Section 9.4, the country is far from the technological frontier). Then any time the incumbent survives a challenge (an event which, as mentioned above may be unobservable), no deviation should be observed from *equal treatment among citizens with political agency*, whether it be achieved by equally redistributing B_t or by implementing the egalitarian public good G_t. Only in the period following a successful challenge can unequal treatment obtain among the citizens with political agency, and then only if k is large. This is the scenario in which the challenger won the power struggle by promising inequitable redistribution, and must now make good on his promises (refer to the discussion following Proposition 2). Observably, this means that after a coup we expect the successful challenger to preferentially reward his supporters among the citizens with political agency, especially if the coup was risky, and the rest (including citizens without political agency) to receive zero. After a time the successful challenger becomes the incumbent, and the rent distribution is expected revert back to equal treatment among citizens with political agency.

10 Conclusions

Little is known theoretically about the political incentives that shape economic policy in non-democracies, and how these might compare to democracies. This paper proposes a new theoretical definition of (non-)democracy based on two "political rights" parameters (η, k) and one "individual right/civil liberties" parameter λ. The parameter λ captures the degree to which individual citizens can expect retribution for their past political activity. I have argued that the parameter λ varies even among democracies, and that this variation captures the distinction between pluralist and consensual democracies that is the subject of a large qualitative literature in political science, and that has not been hitherto formally modeled. Introducing the parameter λ, and studying its effect on policy, is the most innovative aspect of this paper.

The policy space features a trade-off between redistribution and public good provision. This is a classic theme in the political economy tradition of J. Buchanan, G. Tullock, and M. Olson. A new twist here is the distinction between two types of public good: one that delivers egalitarian benefits, the other that delivers non-egalitarian benefits. To my knowledge, this distinction is new in the literature on redistribution vs public good provision. I interpret public goods as growth opportunities that may, or may not, be enabled by government policy. According to this interpretation, my model distinguishes between egalitarian and non-egalitarian growth opportunities.

I find that in any regime with $\lambda < 1$ there is a politico-economic force driving toward equal treatment, but this force is tempered if η is small and k is large. Regimes with $(\eta, k, \lambda) = (1, 0, < 1)$, which I call "quasi-liberal democracies," are the most conducive to equal treatment. The limit point of this class is $(1, 0, 1)$, a regime which I call "liberal democracy." I show that liberal democracy behaves differently from any quasi-liberal democracy because when $\lambda = 1$ citizens do not face a coordination game, but rather a dominant strategy game. As a result, the objective function of politicians is qualitatively different, and it gives politicians incentives to treat citizens inequitably for electoral gain. This qualitative "phase change" is one of the main conceptual insights from this paper. Because non-liberal regimes (including quasi-liberal democracies) have a built-in tendency toward egalitarianism, but liberal democracy has a tendency toward non-egalitarian treatment, quasi-liberal democracies emerge as the best regime for efficient provision of egalitarian public goods, and liberal democracy as the best regime for efficient provision of non-egalitarian public goods.

I have argued that this model has the potential to interpret and reconcile a wide variety of stylized facts. First, that consensual democracies (such as European democracies) have excelled at providing relatively egalitarian "catch-up" growth after WWII, and pluralist democracies (such as the US) has excelled at innovative but relatively non-egalitarian growth. Second, that some non-democracies have adequately provided "catch-up" growth starting from a very low base, but all non-democracies struggle with generating original innovation. Third, that some non-democracies are purely extractive and have no redeeming features.

Future work may illuminate the incentives for political actors, including citizens with political agency, to effect regime transition starting from any given configuration (η, k, λ). Of particular interest, I think, are the incentives to evolve toward larger values of λ, that is, toward stronger protections of individual rights against political retribution.

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A Making the consensual democracy model realistic

Citizens perfectly coordinate on the winner in the equilibrium of my model. Perfect coordination is an unappealing feature in a model that seeks to approximate democratic elections. However, this unappealing feature is easily mitigated. Suppose that λ is close to 1, meaning that supporting the losing politician hardly reduces the voter's material benefits. In this political system, irrespective of the probability of incumbent change, ideological voters do not give up much material benefit by voting their ideology. So one can write a model where a fraction of voters is ideological, and the rest are as in my model. When λ is low the ideological voters will vote their ideology (left or right, for example) but the non-ideological voters will perfectly coordinate on the winning politician provided that they are pivotal, i.e., they can decide the election. In such a game we will have votes for both politicians in equilibrium, which is more realistic.

A second dimensions in which the consensual democracy model can be made more realistic is the incumbent replacement condition. The model assumes that the challenger wins if condition (4) holds, but this is not the same as $a = \int_0^1 a_i \; di \geq 1/2$, which is that the challenger wins if his vote share exceeds 1/2. Helpfully, Proposition 1 by Szkup (2020) shows that the equilibrium analysis in Section 3 extends verbatim if I assume that incumbent replacement happens when

$$R(\theta, a) \leq 0,$$

where R is *any smooth and strictly decreasing function in θ and a* with the property that $R\left(\overline{\theta}, 0\right) < 0$ (the regime may change even if no citizen supports the challenger) and $R\left(\underline{\theta}, 1\right) > 0$ (the incumbent may survive even if all citizens protest). This result is helpful because a function R^* with these properties can be found that lies as close as we wish to the function

$$\widehat{R}(\theta, a) = \frac{1}{2} - a,$$

except for a set of arbitrarily small measure. The function $\widehat{R}$ expresses the democratic replacement rule. By Szkup's (2020) result, the voters' equilibrium strategies in my game under R^* are as specified in Section 3. Furthermore, since the function R^* is decreasing in a, in my game under R^* the autocrat will seek to minimize a and the challenger to maximize it. In sum, the entire analyis developed in this paper goes through if we replace condition (4) with the condition $R^*(\theta, a) \leq 0$, which is a rule for incumbent replacement that closely approximates the democratic rule.

B Proofs and ancillary results

Proof of Lemma 1.

Proof. Citizen i's payoff is:

	Regime change	Status quo
Citizen i supports challenger	$\omega_i - k$	$-k$
Citizen i supports incumbent	0	α_i

Subtracting α_i from the right-hand side column does not alter the citizen's incentives, so we get:

	Regime change $(a \geq 1-\theta)$	Status quo $a < 1-\theta$
Citizen i supports challenger	$\omega_i - k$	$-\alpha_i - k$
Citizen i supports incumbent	0	0

For notational convenience we set

$$b_i = (\omega_i + \alpha_i), c_i = (\alpha_i + k),$$

so that we get:

	Regime change $(a \geq 1-\theta)$	Status quo $(a < 1-\theta)$
Citizen i supports challenger	$b_i - c_i$	$-c_i$
Citizen i supports incumbent	0	0

Now partition citizens into equally treated groups, so that all members of a group g receive the same b_g, c_g. In this setting, Sakovics and Steiner (2012, Proposition 1) show that, in equilibrium, group g supports the challenger if and only if $z_i \geq z_g^*$, as $\sigma \to 0$, all thresholds converge to a common limit $\theta^* = \sum_g m_g \frac{c_g}{b_g}$, so that incumbent survives if and only if:

$$\theta < \sum_g m_g \frac{c_g}{b_g}. \tag{11}$$

This formula, however, requires $b_g > c_g$ (this is a maintained assumption in Sakovics and Steiner 2012). If this condition is violated for some group g' then that group supports the incumbent for sure (dominant strategy). Lemma 1 claims that when $b_g \leq c_g$ is permitted, the equilibrium condition for incumbent survival is:

$$\begin{aligned}\theta - 1 &< \sum_g m_g \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right] \\ &= \sum_g m_g \cdot \left(\frac{\alpha_g + k}{\omega_g + \alpha_g} - 1\right) \cdot \mathbf{1}\left[\omega_g \geq k\right].\end{aligned}$$

To derive this condition observe that if $b_{g'} \leq c_{g'}$ for some group g' then that group does not revolt for sure. In that case, we can eliminate group g' from the game, and there is a new game with new weights

$$\widetilde{m}_g = \frac{m_g}{\sum_{g \neq g'} m_g}.$$

Let's express the condition on behavior for incumbent survival (same as in the old game) using the new-game notation. The condition on behavior using the old notation is:

$$\begin{aligned}\sum_{g \neq g'} m_g a_g &\leq 1 - \theta \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq \frac{1 - \theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq \frac{\sum_{g \neq g'} m_g - \sum_{g \neq g'} m_g + 1 - \theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \frac{\sum_{g \neq g'} m_g - 1 + \theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \frac{-m_{g'} + \theta}{\sum_{g \neq g'} m_g} \\ \sum_{g \neq g'} \widetilde{m}_g a_g &\leq 1 - \widetilde{\theta}.\end{aligned}$$

The condition on behavior for incumbent survival in the new game involves the transformed random variable $\widetilde{\theta}$. Plug into the Sakovics-Steiner condition (11) to get the equilibrium condition (on primitives, not on behavior) for incumbent survival in the new game:

$$
\begin{aligned}
\frac{-m_{g'}+\theta}{\sum_{g\neq g'} m_g} &< \sum_{g\neq g'} \widetilde{m}_g \frac{c_g}{b_g} \\
-m_{g'}+\theta &< \sum_{g\neq g'} m_g \frac{c_g}{b_g} \\
\theta &< m_{g'}\cdot 1+\sum_{g\neq g'} m_g \frac{c_g}{b_g}
\end{aligned}
$$

So, letting g' index any group such that $b_g \leq c_g$, the equilibrium condition for survival (now back in old game notation) is:

$$
\begin{aligned}
\theta &< \sum_g m_g \cdot \mathbf{1}\left[b_g < c_g\right] + \sum_g m_g \cdot \left(\frac{c_g}{b_g}\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= \sum_g m_g \cdot \left\{1 - \mathbf{1}\left[b_g \geq c_g\right] + \left(\frac{c_g}{b_g}\right) \mathbf{1}\left[b_g \geq c_g\right]\right\} \\
&= \sum_g m_g \cdot \left\{1 + \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right]\right\} \\
&= \left(\sum_g m_g\right) + \sum_g m_g \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= 1 + \sum_g m_g \left(\frac{c_g}{b_g} - 1\right) \mathbf{1}\left[b_g \geq c_g\right] \\
&= 1 + \sum_g m_g \cdot \left(\frac{\alpha_g + k}{\omega_g + \alpha_g} - 1\right) \cdot \mathbf{1}\left[\omega_g \geq k\right].
\end{aligned}
$$

Note that the condition reduces to the Sakovics-Steiner condition (11) when $b_g > c_g$. Rearranging the above inequality we get the following expression for the equilibrium condition for survival:

$$
\begin{aligned}
1-\theta &> \sum_g m_g \left(1 - \frac{\alpha_g + k}{\omega_g + \alpha_g}\right) \cdot \mathbf{1}\left[\omega_g \geq k\right] \\
&= \underbrace{\sum_g m_g \left(\frac{\omega_g - k}{\omega_g + \alpha_g}\right) \cdot \mathbf{1}\left[\omega_g \geq k\right]}_{\text{incumbent vulnerability index}}.
\end{aligned}
$$

■

Lemma 2 ***(characterizing*** $\overline{v}$***)*** *The concave envelope* $\overline{v}$ *of the function* v *defined in (8) has the following form:*

$$\overline{v}(\alpha,\omega;k) = \begin{cases} \frac{1}{\left(\sqrt{(\alpha+k)}+\sqrt{k}\right)^2}\cdot\omega & \text{for } \omega < h(\alpha;k) \\ v(\alpha,\omega;k) & \text{for } \omega \geq h(\alpha;k) \end{cases},$$

where

$$h(\alpha;k) = k + \sqrt{k\alpha + k^2}. \tag{12}$$

Proof. Compute the derivative at any point ω:

$$\frac{dv}{d\omega}(\alpha,\omega;k) = \frac{\alpha+k}{(\omega+\alpha)^2}\cdot \mathbf{1}[\omega \geq k]. \tag{13}$$

Now compute the slope r_g of the ray going through any $v(\alpha,\omega;k)$ with $\omega > k$:

$$r_\alpha = \frac{v(\alpha,\omega;k)}{\omega} = \frac{1}{\omega}\left(1-\frac{\alpha+k}{\omega+\alpha}\right). \tag{14}$$

At the tangency point $\omega = h(\alpha;k)$ it must be $v' = r_\alpha$. Use this condition to solve for $h(\alpha;k)$:

$$\begin{aligned} \frac{\alpha+k}{(h(\alpha;k)+\alpha)^2} &= \frac{1}{h(\alpha;k)}\left(1-\frac{\alpha+k}{h(\alpha;k)+\alpha}\right) \\ \frac{\alpha+k}{(h(\alpha;k)+\alpha)^2} &= \frac{1}{h(\alpha;k)}\left(\frac{h(\alpha;k)-k}{h(\alpha;k)+\alpha}\right) \\ \frac{\alpha+k}{(h(\alpha;k)+\alpha)} &= \frac{1}{h(\alpha;k)}(h(\alpha;k)-k) \\ (\alpha+k)h(\alpha;k) &= (h(\alpha;k)-k)(h(\alpha;k)+\alpha) \end{aligned}$$

Solving for $h(\alpha;k)$ yields two solutions: $k \pm \sqrt{k\alpha+k^2}$, but we are looking for the one exceeding k, so the relevant solution is:

$$h(\alpha;k) = k + \sqrt{k\alpha + k^2}.$$

The slope r_g is:

$$\begin{aligned} r_g &= v'\left(h\left(\alpha;k\right);\alpha\right) \\ &= \frac{\alpha+k}{\left(h\left(\alpha;k\right)+\alpha\right)^2} \\ &= \frac{\alpha+k}{\left(\alpha+k+\sqrt{k\alpha+k^2}\right)^2} \\ &= \frac{\alpha+k}{\left(\alpha+k+\sqrt{k}\sqrt{\alpha+k}\right)^2} \\ &= \frac{1}{\left(\sqrt{(\alpha+k)}+\sqrt{k}\right)^2}. \end{aligned}$$

■

Proof of Proposition 1.

Proof. For uniqueness, see Lemma 3.

First, suppose $\eta < 1$. No rational politician would make any positive promises to powerless citizens. This proves part 1.

Now I state the politicians' problems when (η, λ) does not necessarily equal $(1, 0)$. Refer back to matrix (3), and subtract $\lambda\alpha_i$ from the left-hand column and $\lambda\omega_i$ from the right-hand one. This does not alter the citizen's incentives, and results in:

$$\begin{array}{lcc} & \text{Regime change} & \text{Status quo} \\ \text{Citizen } i \text{ supports challenger} & \left(1-\lambda\right)\omega_i - k & -k \\ \text{Citizen } i \text{ supports incumbent} & 0 & \left(1-\lambda\right)\alpha_i \end{array} \tag{15}$$

This game is strategically equivalent to the case $\lambda = 0$ that was analyzed in Lemma 1 except that here: the politician's control variables are $x_i = (1-\lambda)\,\alpha_i$ and $y_i = (1-\lambda)\,\omega_i$; and, also, the mass of citizens with political agency is $\eta < 1$. The latter difference is strategically irrelevant for voters because, given a game where the set of players is $H \subset [0, 1]$, payoffs are given by (15), and the condition for regime change is (4), one can define a strategically equivalent "replica game" where the set of players is $[0, 1]$, payoffs are still given by (15), and the condition for regime change is now:

$$\int_0^1 a_i \, di \geq \frac{1-\theta}{\eta}.$$

Only two properties of the distribution of θ are required to get the right-hand side in (5). First, the random variable $1-\theta$ must be uniformly distributed and also the interval $\left[\left(1-\overline{\theta}\right),\left(1-\underline{\theta}\right)\right]$ is a superset of $[0,1]$. Since this has been assumed already, it follows that for any $\eta \in (0,1]$, $(1-\theta)/\eta$ is uniformly distributed and, furthermore, the interval $\left[\left(1-\overline{\theta}\right)/\eta,\left(1-\underline{\theta}\right)/\eta\right]$ is a superset of $[0,1]$. Therefore the replica game satisfies all the conditions required by Lemma 1, and it follows that the citizens' equilibrium behavior in the replica game is described by Lemma 1 except that $\{\boldsymbol{\alpha},\boldsymbol{\omega}\}$ are replaced by $\{\mathbf{x},\mathbf{y}\}$. Therefore in the original game the incumbent seeks to minimize:

$$\int\limits_H \frac{y_i-k}{y_i+x_i}\cdot \mathbf{1}\left[y_i \geq k\right]\ di. \tag{16}$$

With the change of variables, and taking account of the fact that citizens without political agency must receive zero, the incumbent's budget constraint (1) rewrites as:

$$\int\limits_H \frac{x_i}{(1-\lambda)}\ di \leq \frac{B_1}{\eta},$$

Multiplying through by $(1-\lambda)$ yields:

$$\int\limits_H x_i\ di \leq \overline{B}_1.$$

The challenger's problem is symmetric.

Incumbent's best response: In either case 3 or 4, the challenger's strategy may be described as follows. The challenger sets $y_i = y^*$ with probability p independent of i, and $y_i = 0$ with probability $(1-p)$. Using expression (8) for v we may write the incumbent's problem as:

$$\min_{\mathbf{x}} \int\limits_H p\cdot v\left(x_i, y^*; k\right)\ di. \tag{17}$$

$$\text{s.t.} \int\limits_H x_i\ di \leq \overline{B}_1.$$

The function v is symmetric and strictly convex in $\mathbf{x}$ because $y^* = h\left(\overline{B}_1; k, \lambda\right) > k$ (refer to expression 8), so the solution to problem (17) is $x_i^* = \overline{B}_1$ for all $i \in H$, or $\alpha_i^* = B_1/\eta$.

Challenger's best response. The challenger maximizes incumbent vulnerability, i.e., expression (16), given $x_i^* = \overline{B}_1$ for all i. Using expression (8) for v we may write the challenger's problem as:

$$\max_{\mathbf{y}} \int_H v\left(\overline{B}_1, y_i; k\right) \; di \tag{18}$$
$$\text{s.t.} \int_H y_i \; di \le \overline{B}_2.$$

Let $\overline{v}(\alpha, \omega)$ denote the concave envelope of $v(\alpha, \omega)$ (refer to Figure 3). The following problem

$$\max_{\mathbf{y}} \int_H \overline{v}\left(\overline{B}_1, y_i; k\right) \; di \tag{19}$$
$$\text{s.t.} \int_H y_i \; di \le \overline{B}_2.$$

is a relaxed version of problem (18) because $\overline{v}\left(\overline{B}_1, y; k\right) \ge v\left(\overline{B}_1, y; k\right)$. Because the objective function in problem (19) is symmetric and concave in $\mathbf{y}$, the problem's solution is $y_i = \overline{B}_2$ for all i. Therefore the value of the relaxed problem must be $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right)$.

In case 3 $\overline{B}_2 \ge h\left(\overline{B}_1; k\right)$ implies $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right) = v\left(\overline{B}_1, \overline{B}_2; k\right)$ (refer to Lemma 2). Therefore the value of the relaxed problem (19) is achievable in the original problem (18) by setting $y_i^* = \overline{B}_2$ for all $i \in H$. This implies that $y_i^* \equiv \overline{B}_2$, or $\omega_i^* = B_2/\eta$ for all $i \in H$, is the solution to the original problem (18).

In case 4 $\overline{B}_2 < h\left(\overline{B}_1; k\right)$ implies $\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right) > v\left(\overline{B}_1, \overline{B}_2; k\right)$, and so the value of the relaxed problem (19) is *not* achievable in the original problem (18) by setting $y_i^* = \overline{B}_2$ for all i. By construction of the concave envelope we have:

$$\overline{v}\left(\overline{B}_1, \overline{B}_2; k\right) = v\left(\overline{B}_1, h\left(\overline{B}_1; k\right)\right) \cdot \frac{\overline{B}_2}{h\left(\overline{B}_1; k\right)}, \tag{20}$$

where $h(\alpha; k)$ is as in expression (6) in light of Lemma 2. Expression (20) shows that the value of the relaxed problem is achievable in the original problem (18) by promising $y_i^* = h\left(\overline{B}_1; k\right)$, or $\omega_i^* = h\left(\overline{B}_1; k\right)/(1-\lambda)$ to a mass $\overline{B}_2/h\left(\overline{B}_1; k\right)$ of the citizens with political agency, and $y_i^* = 0$ to the rest. ■

Proof of Proposition 2

Proof. For uniqueness, see Lemma 3.

Part 1. Obvious.

Part 2. Observe that the incumbent does not take advantage of the targetability of redistribution (see Proposition 1), therefore the incumbent will promise the agent-optimal policy.

Part 3. Given that the incumbent's strategy is $x_i^* = M$, by the same logic as in the proof of Proposition 1 the challenger's is: $y_i^* = M$ to everyone if $v(M; M) \geq \overline{v}\left(M, \overline{B}\right)$, else he will redistribute the budget unequally. Condition (19) holds for any $M \geq \overline{B}$ when k is small because when $k = 0$ we have $\overline{v}(\alpha, \omega; 0) = v(\alpha, \omega; 0)$ for any (α, ω) and the function $v(\alpha, \omega; 0)$ is increasing in ω; the desired result then follows by continuity in k of the functions v and $\overline{v}$. Finally, for part 4 fix B and let us look for parameter constellations such the challenger promises inequitable redistribution even though $\overline{G} > \overline{B}$. First, let us set k large enough that $\overline{B} < h\left(\overline{B}, k\right)$. We then have $\overline{B} < h\left(\overline{G}; k\right) = h(M; k)$ for any $\overline{G} > \overline{B}$. Finally, refer to Figure 3: in the region $\omega < h(\alpha, k)$ any two values of ω sufficiently close to each other violate $v \geq \overline{v}$. As any choice of $\overline{G} > \overline{B}$ sufficiently close to $\overline{B}$ lies within the region $\omega < h(M, k)$, this choice of $\overline{G}$ produces a violation of condition (9). This means that the challenger's best response is to redistribute B unequally.

Part 5. The incumbent promises the public good iff $G > B/\eta$, and the pairs (G, B) that satisfy this inequality grows as η increases. The challenger promises the public good if, simultaneously, $G \geq B/\eta$ (else redistribution strategically dominates the public good) and condition (9) holds. When $G \geq B/\eta$ holds condition (9) reads:

$$v\left(\overline{G}, \overline{G}; k\right) \geq \overline{v}\left(\overline{G}, \overline{B}; k\right) = \frac{v\left(\overline{G}, h\left(\overline{G}; k\right)\right)}{h\left(\overline{G}; k\right)} \overline{B}. \tag{21}$$

This inequality depends on η only through $\overline{B} = (1 - \lambda) B/\eta$. As η increases the pairs (G, B) that satisfy both inequalities grows.

Part 6. The incumbent promises the public good iff $G > B/\eta$, which is independent of λ. The challenger promises the public good if, simultaneously, $G > B/\eta$ (which is independent of λ) and if condition (21) holds. The right hand side of (21) is strictly positive for every $\lambda < 1$. The left hand side equals zero whenever $\mathbf{1}\left[\overline{G} \geq k\right] = 0$. Therefore, condition (21) fails whenever $k > \overline{G} = (1 - \lambda) G$. This condition rewrites as $\lambda > (G - k)/G$.
■

Lemma 3 *There is a unique equilibrium in symmetric strategies in Propositions 1 and 2.*

Proof. The proof deals with the case $(\eta, \lambda) = (1, 0)$. The case $(\eta, \lambda) = (1, 0)$ is a straightforward extension.

Take any equilibrium in which the challenger uses the symmetric strategy where promises are drawn from the distribution F_2. Expression (7) reads:

$$\begin{aligned} & \int v(\alpha_i, \omega_i; k)\; di \\ = & \int\int v(\alpha_i, \omega; k)\; dF_2(\omega)\; di \\ = & \int Q(\alpha_i; k)\; di, \end{aligned} \tag{22}$$

where the function

$$Q(\alpha_i; k) = \int v(\alpha_i, \omega; k)\; dF_2(\omega)$$

is convex in α_i, and indeed strictly so because rationality requires F_2 placing positive probability on some $\omega > k$. Therefore, the problem of minimizing (22) subject to the incumbent's budget constraint (1) yields $\alpha_i = \alpha$ for all i. Hence, in any equilibrium where the challenger uses a symmetric strategy F_2, the incumbent uses the symmetric strategy $\alpha_i \equiv \max[B, G]$. Now, the challenger's best response to this strategy is unique and symmetric, as shown in the proof of Propositions 1 and 2. Therefore, the equilibrium in Proposition 1 is the unique equilibrium in symmetric strategies. ■

Proof of Theorem 7

Proof. Part 1. Obvious because $\widetilde{G}$ is dominated by redistribution.

Part 2: not an equilibrium for both politicians to provide the agent-optimal public good.

Suppose both politicians promise the public good. Then the vulnerability index is:

$$\begin{aligned} & \int_H \int_0^{\overline{g}} \frac{(1-\lambda)g - k}{(1-\lambda)g + (1-\lambda)g} \cdot \mathbf{1}\left[(1-\lambda)g \geq k\right]\; \frac{1}{\overline{g}} dg\; di \\ = & \; \frac{\eta}{\overline{g}}\frac{1}{2}\int_{\overline{k}}^{\overline{g}} \frac{g - \overline{k}}{g}\; dg, \end{aligned} \tag{23}$$

where we denote $\overline{k} = k/(1-\lambda)$ If the incumbent deviates to equal redistribution $\alpha_i \equiv \frac{B}{\eta}$ the vulnerability index is:

$$\begin{aligned} & \int\limits_H \int_0^{\overline{g}} \frac{g-k}{g+\left(\frac{B}{\eta}\right)} \cdot \mathbf{1}\left[g \geq \overline{k}\right] \ \frac{1}{\overline{g}} dg \ di \\ = \ & \frac{\eta}{\overline{g}} \int_{\overline{k}}^{\overline{g}} \frac{g-\overline{k}}{g+\left(\frac{B}{\eta}\right)} \ dg. \end{aligned} \tag{24}$$

We seek values of $\overline{g}$ such that the deviation is profitable, that is, such that (23) is larger than (24) (recall that the incumbent seeks to minimize vulnerability), that is:

$$\frac{1}{2} \int_{\overline{k}}^{\overline{g}} \frac{g-\overline{k}}{g} \ dg > \int_{\overline{k}}^{\overline{g}} \frac{g-\overline{k}}{g+\left(\frac{B}{\eta}\right)} \ dg. \tag{25}$$

For given parameters n, b, the following antiderivative formula is known:

$$\int \frac{x+n}{x+b} \ dx = x + (n-b) \ln|x+b| + C.$$

Using this formula, (25) can be written as:

$$\frac{1}{2}\left\{\left[\overline{g} - \overline{k}\ln(\overline{g})\right] - \left[\overline{k} - \overline{k}\ln\left(\overline{k}\right)\right]\right\} > \left[\overline{g} + \left(-\overline{k} - \frac{B}{\eta}\right)\ln\left(\overline{g} + \frac{B}{\eta}\right)\right] - \left[\overline{k} + \left(-\overline{k} - \frac{B}{\eta}\right)\ln\left(\overline{k} + \frac{B}{\eta}\right)\right]. \tag{26}$$

Express $\overline{g}$ as the following monotone transformation of the ancillary parameter C:

$$\overline{g} = (C-1)\frac{B}{\eta} + C\overline{k}. \tag{27}$$

Substitute this expression for $\overline{g}$ into (26) and perform some algebra (see Appendix C) to get:

$$2\left(\frac{B}{\eta\overline{k}} + 1\right)\left[\ln(C) - \frac{(C-1)}{2}\right] > \ln\left(C + (C-1)\frac{B}{\eta\overline{k}}\right) \tag{28}$$

The term in brackets on the LHS is a single-peaked function of C that is positive if:

$$\ln(2c+1) > c,$$

where $c = (C-1)/2$. This is the case for $c \in (0, 1.2564)$. In this interval, both sides of inequality (28) go to infinity as $\overline{k} \to 0$ or $\eta \to 0$ but the LHS grows faster, so for $\overline{k}$ or

η small enough inequality (28) holds. Within this interval the set of c's that make the public good agent-optimal is that which satisfies $\overline{g}/2 > \frac{B}{\eta}$; substitute

$$\frac{\overline{g}}{2} = c\left(\frac{B}{\eta}\right) + \left(c + \frac{1}{2}\right)\overline{k} \tag{29}$$

from (27) and isolate C to get:

$$c > \frac{2B - \eta\overline{k}}{2B + 2\eta\overline{k}}, \tag{30}$$

which is satisfied for any $\left(\eta, \overline{k}\right)$ if $c > 1$. Therefore, if $c \in (1, 1.2564)$ the public good is agent-optimal but, for $\overline{k}$ or η small enough, it is not an equilibrium for both politicians to provide it. Use (29) to characterize the values of the public good with the desired property. This is the set of all $\widetilde{G}$'s such that:

$$\mathbb{E}\left(\widetilde{G}\right) = \frac{\overline{g}}{2} = c\left(\frac{B}{\eta}\right) + \left(c + \frac{1}{2}\right)\overline{k} \text{ for } c \in (1, 1.2564)\,.$$

For $k \to 0$ this set converges to the set:

$$\left\{\widetilde{G} : \mathbb{E}\left(\widetilde{G}\right) = c\frac{B}{\eta} \text{ for } c \in (1, 1.2564)\right\}.$$

Part 2: equilibrium with egalitarian redistribution

Incumbent's best response. Suppose the challenger promises $\omega_i = B/\eta$ for all $i \in H$. The vulnerability index is:

$$\int_H \frac{\left(\frac{B}{\eta}\right) - \overline{k}}{\left(\frac{B}{\eta}\right) + \alpha_i} \cdot \mathbf{1}\left[\frac{B}{\eta} \geq \overline{k}\right] \, di.$$

This is a symmetric and strictly convex function of $\boldsymbol{\alpha}$ for $\frac{B}{\eta} \geq \overline{k}$. If the incumbent is restricted to using redistribution, then her best response is to set $\alpha_i = B/\eta$ for all $i \in H$. The value of the incumbent's problem assuming $\frac{B}{\eta} \geq \overline{k}$ is:

$$\begin{aligned} &\int_H \frac{1}{2}\frac{\left(\frac{B}{\eta}\right) - \overline{k}}{\left(\frac{B}{\eta}\right)} \, di \\ = \;& \frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right) - \overline{k}}{\left(\frac{B}{\eta}\right)} \end{aligned} \tag{31}$$

Now remove the restriction: would the incumbent benefit from promising $\widetilde{G}$? Assuming $\frac{B}{\eta} \geq \overline{k}$ the vulnerability index after a deviation to $\widetilde{G}$ would read:

$$\int_H \int_0^{\overline{g}} \frac{\left(\frac{B}{\eta}\right) - \overline{k}}{\left(\frac{B}{\eta}\right) + g} \frac{1}{\overline{g}} dg \; di. \tag{32}$$

After some algebra (see Appendix C) we get that the vulnerability after a deviation is larger than that under the posited equilibrium strategy, i.e., (32) is larger than (31) if and only if:

$$\mathbb{E}\left(\widetilde{G}\right) = \overline{g}/2 < c\left(\frac{B}{\eta}\right) \tag{33}$$

where $c \approx 1.26$ is the solution to $c = \log(1 + 2c)$.

Challenger's best response. Suppose the incumbent promises $\alpha_i = B/\eta$ for all $i \in H$. The vulnerability index is:

$$\int_H \frac{\omega_i - \overline{k}}{\omega_i + \left(\frac{B}{\eta}\right)} \cdot \mathbf{1}\left[\omega_i \geq \overline{k}\right] \; di.$$

This is a symmetric and strictly concave function of $\boldsymbol{\omega}$ for $\frac{B}{\eta} \geq \overline{k}$. Suppose the challenger is restricted to using redistribution. Then his best response does no worse than setting $\alpha_i = B/\eta$ for all $i \in H$, and the value of the challenger's problem is greater or equal than (31). Now remove the restriction: would the challenger benefit from promising $\widetilde{G}$? The vulnerability index would read:

$$\int_H \int_{\overline{k}}^{\overline{g}} \frac{g - \overline{k}}{g + \left(\frac{B}{\eta}\right)} \frac{1}{\overline{g}} dg \; di,$$

which is exactly equal to (24). After some algebra (see Appendix C) we get that (31) is greater than (24), and so the challenger would not benefit from promising $\widetilde{G}$, if and only if

$$\mathbb{E}\left(\widetilde{G}\right) = \overline{g}/2 < c\left(\frac{B}{\eta}\right) \tag{34}$$

where $c \approx 1.26$ is the solution to $c = \ln(1 + 2c)$.

Part 3. Suppose candidate j plays the prescribed equilibrium strategy, i.e., a uniform $[0, m]$ where $m = \max[\overline{g}, 2B/\eta]$. I will show that candidate $-j$'s best response is to promise the distribution with the highest possible mean, regardless of the specific shape of that distribution. Therefore a best response to j's prescribed equilibrium strategy is to play the public good or redistribution, whichever is more efficient, which is in fact $-j$'s prescribed equilibrium strategy.

If candidate j draws her promises from $U[0, m]$ and candidate $-j$ from a probability distribution X_{-j}, candidate $-j$'s vote share is:

$$\begin{aligned} S_{-j} &= \int_0^m \frac{x}{m}\, dF_{-j}(x) \\ &\leq \frac{1}{m}\mathbb{E}(X_{-j}) \end{aligned}$$

where $F_{-j}(x)$ represents the probability that X_{-j} is less than or equal to x, and equality holds when $X_{-j} \leq m$. This concludes the proof. ■

C Appendix not for publication: calculations for the proof of Theorem 7

From (26) to (28)

Due to our choice of $\overline{g}$ as in (27) we have

$$\left(\overline{g}+\frac{B}{\eta}\right)=C\left(\overline{k}+\frac{B}{\eta}\right).$$

Substitute into (26) to get:

$$\begin{aligned}
\frac{1}{2}\left(\overline{g}-\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\left[\ln\left(\overline{g}+\frac{B}{\eta}\right)-\ln\left(\overline{k}+\frac{B}{\eta}\right)\right] &< \frac{1}{2}\left\{-\overline{k}\left[\ln\left(\overline{g}\right)-\ln\left(\overline{k}\right)\right]\right\}\\
\frac{1}{2}\left(\overline{g}-\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\frac{1}{2}\left(\left(C-1\right)\frac{B}{\eta}+C\overline{k}-\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\frac{1}{2}\left(\left(C-1\right)\frac{B}{\eta}+\left(C-1\right)\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\frac{\left(C-1\right)}{2}\left(\frac{B}{\eta}+\overline{k}\right)-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(C\right) &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\left(\frac{B}{\eta}+\overline{k}\right)\left[\frac{\left(C-1\right)}{2}-\ln\left(C\right)\right] &< \frac{1}{2}\left\{-\overline{k}\ln\left(C+\frac{\left(C-1\right)}{\overline{k}}\frac{B}{\eta}\right)\right\}\\
\left(\frac{B}{\eta\overline{k}}+1\right)\left[\frac{\left(C-1\right)}{2}-\ln\left(C\right)\right] &< -\frac{1}{2}\left\{\ln\left(C+\left(C-1\right)\frac{B}{\eta\overline{k}}\right)\right\}\\
\left(\frac{B}{\eta\overline{k}}+1\right)\left[\ln\left(C\right)-\frac{\left(C-1\right)}{2}\right] &> \frac{1}{2}\left\{\ln\left(C+\left(C-1\right)\frac{B}{\eta\overline{k}}\right)\right\}\\
2\left(\frac{B}{\eta\overline{k}}+1\right)\left[\ln\left(C\right)-\frac{\left(C-1\right)}{2}\right] &> \ln\left(C+\left(C-1\right)\frac{B}{\eta\overline{k}}\right)
\end{aligned}$$

Getting expression (30)

$$
\begin{aligned}
c\left(\frac{B}{\eta}\right)+\left(c+\frac{1}{2}\right)\overline{k} &> \frac{B}{\eta} \\
c\left(\frac{B}{\eta}\right)+c\overline{k} &> \frac{B}{\eta}-\frac{1}{2}\overline{k} \\
c\left(\frac{B}{\eta}+\overline{k}\right) &> \frac{B}{\eta}-\frac{1}{2}\overline{k} \\
c\left(\frac{B+\eta\overline{k}}{\eta}\right) &> \frac{2B}{2\eta}-\frac{\eta\overline{k}}{2\eta} \\
c &> \frac{2B-\eta\overline{k}}{2B+2\eta\overline{k}}
\end{aligned}
$$

Getting to (??)

The vulnerability index after the deviation reads:

$$
\begin{aligned}
& \frac{1}{\overline{g}}\int_H\int_0^{\overline{g}}\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)+\alpha}\,d\alpha di \\
=\; & \frac{1}{\overline{g}}\eta\int_0^{\overline{g}}\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)+\alpha}\,d\alpha \\
=\; & \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\int_0^{\overline{g}}\frac{1}{\left(\frac{B}{\eta}\right)+\alpha}\,d\alpha \\
=\; & \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\int_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}}\frac{1}{x}\,dx \\
=\; & \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\left[\ln(x)\right]_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}}.
\end{aligned}
$$

So the incumbent prefers equal redistribution to the public good iff the vulnerability at the public good is larger, that is, if

$$
\begin{aligned}
\frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)} &< \frac{1}{\overline{g}}\eta\left[\left(\frac{B}{\eta}\right)-\overline{k}\right]\left[\log\left(x\right)\right]_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}} \\
\frac{1}{2}\frac{1}{\left(\frac{B}{\eta}\right)} &< \frac{1}{\overline{g}}\left[\ln\left(x\right)\right]_{\frac{B}{\eta}}^{\left(\frac{B}{\eta}\right)+\overline{g}} \\
\frac{\overline{g}}{2}\frac{1}{\left(\frac{B}{\eta}\right)} &< \ln\left(\frac{B}{\eta}+\overline{g}\right)-\ln\left(\frac{B}{\eta}\right)
\end{aligned}
$$

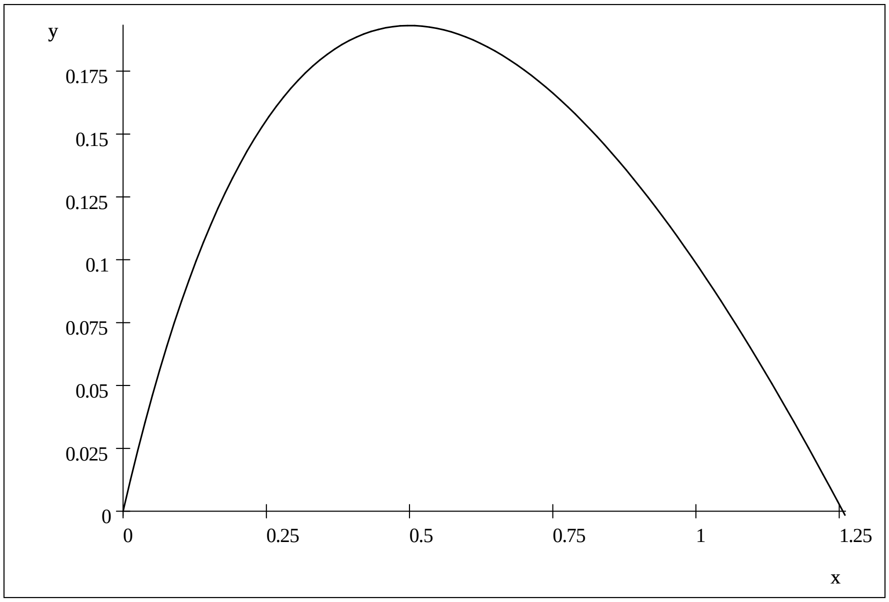
We need

$$
\begin{aligned}
\frac{\overline{g}}{2}\frac{1}{\left(\frac{B}{\eta}\right)} &< \ln\left(\frac{B}{\eta}+\overline{g}\right)-\ln\left(\frac{B}{\eta}\right) \\
\frac{\overline{g}}{2} &< \left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}+\overline{g}\right)-\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) \\
\overline{g} &< 2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}+\overline{g}\right)-2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) \\
2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) &< 2\frac{B}{\eta}\ln\left(\frac{B}{\eta}+\overline{g}\right)-\overline{g}
\end{aligned}
$$

Replace $\overline{g}=c2\frac{B}{\eta}$. Then we get:

$$
\begin{aligned}
2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) &< 2\frac{B}{\eta}\ln\left(\frac{B}{\eta}+c2\frac{B}{\eta}\right)-c2\frac{B}{\eta} \\
2\left(\frac{B}{\eta}\right)\ln\left(\frac{B}{\eta}\right) &< 2\frac{B}{\eta}\ln\left(\frac{B}{\eta}\left(1+c2\right)\right)-c2\frac{B}{\eta} \\
\ln\left(\frac{B}{\eta}\right) &< \left[\ln\left(\frac{B}{\eta}\right)+\ln\left(1+2c\right)\right]-c \\
0 &< \log\left(1+2c\right)-c.
\end{aligned}
$$

The function $\log\left(1+2c\right)-c$



is positive if $c \in (0, 1.26)$. After restricting to values of $c \geq 1$ in order to ensure that the public good efficient we get the condition

$$\frac{\overline{g}}{2\left(\frac{B}{\eta}\right)} = \frac{\mathbb{E}\left(\widetilde{G}\right)}{\left(\frac{B}{\eta}\right)} = c \in (1, 1.26).$$

Getting to (34)

We need (31) greater than (24), i.e.:

$$\begin{aligned}
\frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)} &> \frac{\eta}{\overline{g}}\int_{\overline{k}}^{\overline{g}}\frac{g-\overline{k}}{g+\left(\frac{B}{\eta}\right)}\,dg \\
\frac{1}{2}\eta\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)} &> \frac{\eta}{\overline{g}}\left[\overline{g}+\left(-\overline{k}-\frac{B}{\eta}\right)\ln\left(\overline{g}+\frac{B}{\eta}\right)\right]-\left[\overline{k}+\left(-\overline{k}-\frac{B}{\eta}\right)\ln\left(\overline{k}+\frac{B}{\eta}\right)\right] \\
\frac{\overline{g}}{2}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> \overline{g}-\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\overline{g}+\frac{B}{\eta}\right)-\overline{k}+\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\overline{k}+\frac{B}{\eta}\right) \\
\frac{\overline{g}}{2}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> \overline{g}-\overline{k}+\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{\overline{k}+\frac{B}{\eta}}{\overline{g}+\frac{B}{\eta}}\right)
\end{aligned}$$

Due to our choice of $\overline{g}$ as in (27) we have

$$\begin{aligned}\left(\overline{g}+\frac{B}{\eta}\right) &= C\left(\overline{k}+\frac{B}{\eta}\right)\\ \overline{g} &= C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\end{aligned}$$

and so the inequality rewrites as:

$$\begin{aligned}
\frac{C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}}{2}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> \left((C-1)\frac{B}{\eta}+C\overline{k}-\overline{k}\right)+\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{1}{C}\right)\\
\left[C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\right]\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> 2\left[(C-1)\frac{B}{\eta}+C\overline{k}-\overline{k}\right]+2\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{1}{C}\right)\\
\left[C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\right]\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> 2(C-1)\left(\frac{B}{\eta}+\overline{k}\right)+2\left(\overline{k}+\frac{B}{\eta}\right)\ln\left(\frac{1}{C}\right)\\
\left[C\left(\overline{k}+\frac{B}{\eta}\right)-\frac{B}{\eta}\right]\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right] &> 2\left(\frac{B}{\eta}+\overline{k}\right)\left[(C-1)+\ln\left(\frac{1}{C}\right)\right]\\
2\left(\frac{B}{\eta}+\overline{k}\right)\left[\ln(C)-(C-1)\right] &> \frac{B}{\eta}\left[\frac{\left(\frac{B}{\eta}\right)-\overline{k}}{\left(\frac{B}{\eta}\right)}\right]-C\left(\overline{k}+\frac{B}{\eta}\right)\\
\left(\frac{B}{\eta}+\overline{k}\right)\left[2\ln(C)-2(C-1)\right] &> \left[\frac{B}{\eta}-\overline{k}\right]-C\left(\overline{k}+\frac{B}{\eta}\right)\\
\left(\frac{B}{\eta}+\overline{k}\right)\left[2\ln(C)-C+2\right] &> \left[\frac{B}{\eta}-\overline{k}\right]\\
2\ln(C)-C &> -2+\frac{\left[\frac{B}{\eta}-\overline{k}\right]}{\left(\frac{B}{\eta}+\overline{k}\right)}
\end{aligned}$$

Now with the change of variables $c=(C-1)/2$ we get $C=2c+1$:

$$\begin{aligned}2\ln(C) &> C-2+\frac{\left[\frac{B}{\eta}-\overline{k}\right]}{\left(\frac{B}{\eta}+\overline{k}\right)}\\ 2\ln(2c+1) &> 2c-1+\frac{\left[\frac{B}{\eta}-\overline{k}\right]}{\left(\frac{B}{\eta}+\overline{k}\right)},\end{aligned}$$

which for $\overline{k} \to 0$ reduces to the desired inequality:

$$2 \ln (2c + 1) > 2c.$$

When $k = 0$ the right hand side in (5) reduces to the politicians' payoff in the probabilistic voting game studied by, among others, Brams and Davis (1974) and Snyder (1989).[20] Furthermore, while the payoff function (5) is not literally a special case of Lindbeck and Weibull's (1987) probabilistic voting model,[21] the functional form (5) enjoys the convexity/concavity properties that Lindbeck and Weibull's (1987) setting is designed to deliver. Therefore the incentives that shape polician's promises in these probabilistic voting games are the same as in my setting when $\lambda < 1$. This is not to say that the probabilistic voting and my setting are the same in all respects. I pursue the connection between the two settings in Appendix A.

[20] The defining feature of (5) is the Tullock contest function $\omega/(\alpha+\omega)$. Besides voting games, this contest function has also been used in other literatures: see Friedman (1958) and Kovenock and Rojo Arjona (2019).

[21] Lindbeck and Weibull's (1987) framework does not accommodate the Tullock contest function $\omega/(\alpha+\omega)$ featured in equation (5).