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## **Quantifying Partisan Gerrymandering: An Evaluation of the Efficiency Gap Proposal**

Benjamin Plener Cover<sup>1</sup>

### ABSTRACT

*For three decades, the Court has failed to settle on a legal test for partisan gerrymandering, and such claims have uniformly failed – until now. Leveraging a new measure and associated test for partisan gerrymandering called the “efficiency gap,” plaintiffs challenging Wisconsin’s Assembly plan have prevailed before a three-judge federal panel. The measure defines partisan gerrymandering in terms of two parties’ relative efficiency at translating votes for their party into seats in government. The case is now before the Supreme Court, which may embrace the efficiency gap approach and thereby remake the law of electoral districting.*

*Through a synthesis of mathematical and legal analysis, this article examines the efficiency gap’s conceptual premises and real-world performance. The measure may produce counterintuitive results because it favors one democratic norm – partisan fairness – over other norms like electoral competitiveness and the proportionality between votes earned and seats won. A mapmaker can achieve a low efficiency gap by drawing a bipartisan gerrymander that carves up the state into safe seats for incumbents and confers a legislative supermajority on a party that earns only a modest majority of ballots cast. Efforts to promote electoral competitiveness or a closer fit between votes earned and seats won may produce a high efficiency gap. And because the efficiency gap is a single measure based on observed election data, it is vulnerable to manipulation: By suppressing turnout of its competitor’s supporters, a party can artificially reduce a plan’s efficiency gap. The measure’s reliance on definitional choices and stylized assumptions about electoral circumstances limits its appeal as one based on observed election results rather than conjecture and hypotheticals. In light of these normative and methodological concerns, the article concludes that the efficiency gap may be a useful indicator of partisan gerrymandering when appropriately applied, but courts should not adopt it as the exclusive legal definition.*

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<sup>1</sup> Visiting Associate Professor, University of Idaho College of Law. JD, Yale Law School; M.Sc., London School of Economics. For helpful discussions as I developed these ideas, I extend my thanks to Nicholas Stephanopoulos, Eric McGhee, Heather Gerken, Richard Re, Raymond Dacey, Lisa Carlson, Steven Radil, Michele Wiest, and Kenton Bird, as well as participants in the Fifth Annual State & Local Government Law Works-in Progress Conference, the Rocky Mountain Junior Scholars Forum, the Malcolm M. Renfrew Interdisciplinary Colloquium, and the Political Science, Economics, and Geography Working Group at the University of Idaho. For useful feedback during the writing process, I am grateful to Wendy Couture, Kate Evans, David Pimentel, and Sandy Mayson. Kacey Jones, Cody Witko, Justin Bowles, and Kelsey Gooden provided excellent research. As always, I am particularly indebted to Aliza Plener Cover.

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## INTRODUCTION

We may be approaching a watershed moment in the Supreme Court’s gerrymandering jurisprudence. In three cases over the last three decades,<sup>2</sup> political gerrymandering has eluded the Court’s grasp. The Court has unanimously recognized that partisan gerrymandering poses a problem of constitutional significance,<sup>3</sup> but repeatedly fractured on whether and how to intervene. A minority of Justices has insisted that partisan gerrymandering presents a nonjusticiable political question susceptible to no judicially discernible and manageable standard,<sup>4</sup> while a majority of Justices has agreed that political gerrymandering is justiciable but disagreed among themselves about the proper legal standard.<sup>5</sup> Justice Kennedy, the controlling swing vote, has rejected each proposal, while expressing hope that a suitable standard may one day materialize.<sup>6</sup> In the first thirty years after the Court held partisan gerrymandering justiciable, dozens of plaintiffs raised claims of partisan gerrymandering, but not one survived the motions stage<sup>7</sup> – until now.

In a case called *Whitford v. Gill*,<sup>8</sup> plaintiffs have successfully challenged Wisconsin’s Assembly map as a political gerrymander, relying on

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<sup>2</sup> *Davis v. Bandemer*, 478 U.S. 109 (1986); *Vieth v. Jubelirer*, 541 U.S. 267 (2004); *League of United Latin Am. Citizens [LULAC] v. Perry*, 548 U.S. 399 (2006).

<sup>3</sup> *Vieth*, 541 U.S. at 293 (plurality opinion) (“[A]n excessive injection of politics is unlawful.”); *Arizona State Legislature v. Arizona Indep. Redistricting Comm’n*, 135 S. Ct. 2652, 2658 (2015) (“[T]his Court has recognized [that gerrymanders] are incompatible with democratic principles.”) (quoting *Vieth*, 541 U.S. at 292 (plurality opinion)) (internal quotations omitted).

<sup>4</sup> *Bandemer*, 478 U.S. at 144 (O’Connor, J., concurring in the judgment) (joined by Chief Justice Burger and Justice Rehnquist); *Vieth*, 541 U.S. at 271 (plurality opinion) (authored by Justice Scalia and joined by Chief Justice Rehnquist and Justices O’Connor and Thomas); *LULAC*, 548 U.S. at 511 (Scalia, J., concurring in the judgment in part and dissenting in part) (joined by Justice Thomas).

<sup>5</sup> *Bandemer*, 478 at 113 (plurality opinion); *Bandemer*, 478 U.S. at 161 (Powell, J., concurring in part and dissenting in part); *Vieth*, 541 U.S. at 317 (Stevens, J., dissenting); *Vieth*, 541 U.S. at 343 (Souter, J., dissenting); *Vieth*, 541 U.S. at 355 (Breyer, J., dissenting); *Vieth*, 541 U.S. at 306 (Kennedy, J., concurring in the judgment); *LULAC*, 548 U.S. at 447 (2006) (Stevens, J., dissenting in part); *LULAC*, 548 U.S. at 483 (Souter, J., concurring in part and dissenting in part); *LULAC*, 548 U.S. at 491 (Breyer, J., concurring in part and dissenting in part); *LULAC*, 548 U.S. 399, 408 (2006) (Kennedy, J.).

<sup>6</sup> *Vieth*, 541 U.S. at 306 (Kennedy, J., concurring in the judgment); *LULAC*, 548 U.S. at 408.

<sup>7</sup> Nicholas O. Stephanopoulos & Eric M. McGhee, *Partisan Gerrymandering and the Efficiency Gap*, 82 U. CHI. L. REV. 831, 832-833 (2015) (“By our count, claimants’ record over this generation-long period [1986–2015] is roughly zero wins and fifty losses.”); Easha Anand, Comment, *Finding a Path Through the Political Thicket: In Defense of Partisan Gerrymandering’s Justiciability*, 102 CALIF. L. REV. 917, 933 (2014) (“[O]f the thirty-nine decisions surveyed . . . only one found a gerrymander unconstitutional, and that one decision was subsequently dismissed as moot.”); *Vieth*, 541 U.S. at 279-280 (“[I]n all of the cases we are aware of involving that most common form of political gerrymandering, relief was denied.”) (emphasis in original).

<sup>8</sup> 218 F. Supp. 3d 837 (W.D. Wis. 2016).

a newly proposed numeric measure and associated legal test called the “efficiency gap.” In 2014, political scientist Eric McGhee proposed the numeric measure.<sup>9</sup> In 2015, Eric McGhee and leading election law scholar Nicholas Stephanopoulos developed this numeric measure into a legal test specifically designed to address Justice Kennedy’s concerns with prior proposals.<sup>10</sup> In brief, the efficiency gap measure counts the relative number of votes “wasted” by each of two competing political parties; it thereby quantifies the relative efficiency with which each party is able to convert popular support (votes) into governmental power (seats). The legal test classifies as an invalid partisan gerrymandering any plan drawn with discriminatory intent that produces a large, durable, and unjustified efficiency gap. Armed with this new measure and associated legal test, the *Whitford* plaintiffs not only survived the motions stage, but won at trial before a three-judge federal panel.<sup>11</sup> The majority opinion does not endorse wholesale the plaintiffs’ proposal, but extensively discusses the efficiency gap as strong evidence in support of its conclusion that the map was a partisan gerrymander.<sup>12</sup> Wisconsin appealed directly to the Supreme Court, which stayed the panel’s remedial order and ordered full briefing and argument for the 2017-18 term.<sup>13</sup> The *Whitford* case provides Justice Kennedy the

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<sup>9</sup> Eric McGhee, *Measuring Partisan Bias in Single-Member District Electoral Systems*, 39 LEGIS. STUD. Q. 55, 68 (2014).

<sup>10</sup> Stephanopoulos & McGhee, *supra* note 7. Throughout the article, I refer to Eric McGhee and Nicholas Stephanopoulos as the “academic proponents” or simply the “proponents” of the proposed efficiency gap measure and legal test.

<sup>11</sup> *Whitford v. Gill*, 218 F. Supp. 3d 837, 903 (W.D. Wis. 2016). When a plaintiff sues in federal court to challenge the federal constitutionality of an electoral districting plan for a state’s legislature or congressional delegation, federal statutes requires the convening of a three-judge federal panel, with direct appeal to the Supreme Court. 28 U.S.C. § 2284(a); 28 U.S.C. § 1253. See *Cooper v. Harris*, 137 S. Ct. 1455, at 1464 n.2 (2017). See generally *Shapiro v. McManus*, 136 S. Ct. 450 (2015); Joshua A. Douglas, *The Procedure of Election Law in Federal Courts*, 2011 UTAH L. REV. 433 (2011).

<sup>12</sup> *Whitford*, at 903 (“evidence is further bolstered by the plaintiffs’ use of the ‘efficiency gap’...to demonstrate that...their representations rights have been burdened.”); *id.* at 933 (Griesbach, J., dissenting) (“Despite the central role the efficiency gap has played in the case...the majority has declined the Plaintiffs’ invitation to adopt their standard and uses it only as confirming evidence . . .”).

<sup>13</sup> Order Granting Stay, 582 U.S. No. 3:15-cv-00421 (2017). On February 24, 2017, Wisconsin timely filed a notice of appeal in the *Whitford* case. Def.’s Notice of Appeal, Feb. 24, 2017, No. 191. The Supreme Court must reach the merits of a direct appeal from a three-judge panel, either summarily or after full briefing and argument. See 28 U.S.C. § 2284(a); 28 U.S.C. § 1253; U.S. Sup. Ct. R. 18(12). Meanwhile, another federal lawsuit in North Carolina, challenging that state’s congressional districting plan and also relying on the efficiency gap measure, has survived a motion to dismiss. *Common Cause v. Rucho*, 2017 WL 876307, at \*3 (M.D.N.C. Mar. 3, 2017) (opinion denying motions to dismiss). The federal panel has consolidated two cases, one brought by a group of plaintiffs led by Common Cause, the other brought by a group of plaintiffs led by the League of Women Voters. The two groups of plaintiffs make similar legal arguments, but only the League of Women Voters

opportunity to decide whether the efficiency gap approach constitutes the legal test he has been waiting for. If so, the Court may adopt the efficiency gap proposal next term in a blockbuster 5-4 opinion<sup>14</sup> that remakes the law of electoral districting.

As the Court considers *Whitford*, the efficiency gap measure and associated, legal test warrants careful and comprehensive examination. Thus far, the reactions in litigation, scholarship, and popular media have been strong and conflicting.<sup>15</sup> This article contributes to this nascent evaluative

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plaintiffs use the efficiency gap in the discriminatory effect element of the proffered legal test. *Id.* at 3.

<sup>14</sup> Since the Court decided *Vieth* in 2004, Justices Roberts, Alito, Sotomayor, Kagan, and Gorsuch have respectively replaced Justices Rehnquist, O'Connor, Souter, Stevens, and Scalia. Despite this decomposition, Justice Kennedy likely remains in the swing position he has occupied since *Vieth*, because each of these five jurists would likely vote like his or her predecessor on the question of political gerrymandering. *See, e.g.* Kerr v. Hickenlooper, 759 F.3d 1186, 1196 (10th Cir. 2014) (denial of rehearing en banc) (Gorsuch, J. dissenting) (approvingly citing *Vieth* (Scalia, J., plurality)). Justice Kennedy may be disposed to embrace a legal test before further recomposition dislodges him from that swing position.

<sup>15</sup> For scholarly examination of the efficiency gap proposal, see John F. Nagle, *How Competitive Should a Fair Single Member Districting Plan Be?*, 16 ELECTION L.J. \_\_\_\_ (2017); Jonathan S. Krasno, et al., *Can Gerrymanders Be Measured? An Examination of Wisconsin's State Assembly* (May 22, 2016), available at <https://ssrn.com/abstract=2783144>; Edward B. Foley, *Due Process, Fair Play, and Excessive Partisanship: A New Principle for Judicial Review of Election Laws*, 84 U. CHI. L. REV. \_\_\_\_ (forthcoming 2017), available at <https://ssrn.com/abstract=2815892>; Samuel S.-H. Wang, *Three Practical Tests for Gerrymandering: Application to Maryland and Wisconsin*, 15 ELECTION L.J. 367 (2016); Theodore S. Arrington, *A Practical Procedure for Detecting a Partisan Gerrymander*, 15 ELECTION L.J. 385 (2016); Anthony J. McGann, et al, *A Discernable and Manageable Standard for Partisan Gerrymandering*, 14 ELECTION L.J. 295 (2015); Jowei Chen, *Wisconsin Act 43 Analysis*, available at [http://www-personal.umich.edu/~jowei/Wisconsin\\_Act\\_43\\_Analysis.pdf](http://www-personal.umich.edu/~jowei/Wisconsin_Act_43_Analysis.pdf) (last visited Mar. 15, 2017).

The *Rucho* panel briefly discussed the efficiency gap proposal when denying defendant's motion to dismiss. *Rucho*, 2017 WL 876307, at \*3-4. In the *Whitford* panel's post-trial decision on the merits, both the majority and dissent discuss the efficiency gap proposal in depth. *Whitford*, 2016 WL 6837229, at \*50-56; *id.* at 72-78 (Griesbach, J., dissenting). The panel also discussed the proposal when denying Wisconsin's motion to dismiss and motion for summary judgment. Order and Opinion Denying Motion to Dismiss, 3:15-cv-00421-bbc No. 43 (Feb. 17, 2015), available at <http://www.campaignlegalcenter.org/sites/default/files/WvN%2B-%2BMTD%2BDecision.pdf>; Opinion and Order on Motion for Summary Judgment, 3:15-cv-00421-bbc No. 94 (Apr. 7, 2016), available at <http://www.campaignlegalcenter.org/sites/default/files/Opinion%20on%20Motion%20for%20Summary%20Judgment.pdf>.

In addition to the panel's opinions and the parties' briefs, the efficiency gap proposal was analyzed in a declaration and reports submitted by parties' respective experts. The plaintiffs offered two experts – Kenneth R. Mayer, professor of political science at University of Wisconsin-Madison, and Simon Jackman, a professor of political science at Stanford. Wisconsin offered two experts – Sean Trende, a senior election analyst at RealClearPolitics and Nicholas Goedert, a professor of government and law at Lafayette College. Sean Trende submitted an expert declaration.

effort by offering a comprehensive analysis of the proposed efficiency gap measure and legal test. Because the efficiency gap measure stands at the intersection of law and mathematics, this article employs a hybrid methodology of mathematical and legal analysis. When mathematical formulas are infused with legal significance, legal decision-makers must understand the importance and downstream implications of methodological choices and electoral assumptions. The key contribution of this piece is to illuminate the normative significance of the measure's technical properties.

This article evaluates the efficiency gap measure, and its associated legal test, from several analytical frames: correspondence, robustness, and scope. It considers the measure's correspondence to normative judgments about the existence and extent of political gerrymandering along three dimensions. First, *conceptual* correspondence – does defining political gerrymandering as a measure of relative wasted votes correspond to common understandings of the relevant constitutional values at stake in electoral districting? Second, *evaluative* correspondence – does the efficiency gap measure evaluate districting plans in a way that corresponds to our normative assessment of those plans? For example, does the sign of the efficiency gap correspond to our understanding of which party the plan favors? Does the relative gap between two plans correspond to our understanding of which plan constitutes a greater gerrymander? And third, *incentive* correspondence – would the efficiency gap measure, if adopted as the exclusive legal definition for political gerrymandering, incentivize prospective cartographic behavior that is normatively desirable or undesirable?

Related to these questions of normative correspondence are questions of scope and robustness, both of which are also laden with normative content. What is the measure's scope? Can it be applied in the same way in all cases, or are there scenarios in which it applies differently or not at all? Is the measure robust to methodological choices and electoral circumstances, or will the results it produces vary depending on these choices and circumstances? If extant methodological choices will produce different results, a normative question arises: should we choose the methodology that corresponds to normatively desirable outcomes, or the methodology that is most internally coherent? If the results depend on electoral circumstances, the measure may be manipulable by political actors in normatively undesirable ways.

Through an inquiry into the normative correspondence, robustness, and scope of the efficiency gap measure and its associated legal test, I conclude that the most attractive aspect of the efficiency gap – its simplicity – is also the greatest cause for concern. The fundamental appeal of the efficiency gap measure is that it captures in one number something significant about whether, and to what extent, a districting plan is gerrymandered. If utilized as an *indicative* measure of gerrymandering, the efficiency gap measure would be an appropriate and useful aid in many circumstances.

However, in order to reduce partisan gerrymandering to one convenient number, the efficiency gap measure makes considerable sacrifices of correspondence, scope, and robustness. The efficiency gap measure is mono-normative: It privileges one democratic norm – symmetric partisan efficiency – over other relevant, and equally important, democratic norms, including electoral competitiveness, seats-votes proportionality, and voter participation. The efficiency gap measure also relies on a simplistic view of electoral success and the electoral process. It conceptualizes voter influence in a binary fashion based only on whether ballots cast directly contributed to a party's victory. It does not take into account other forms of voter influence, nor does it take into account electoral dynamics such as voter suppression that constrain the number of ballots cast.

By oversimplifying the electoral process and the multiple democratic norms it implicates, the efficiency gap measure misses critical information and ignores critical normative questions. As a result, the measure poses normative risks and incentivizes normatively problematic behavior. For example, the measure can be manipulated through voter suppression, it permits and even encourages uncompetitive elections, and it idealizes districting plans and election results in which a party with sufficient vote share attains all of the seats. Thus, although the efficiency gap measure may be usefully employed as an indicative measure, the Supreme Court should not embrace it as an exclusive, definitional measure of gerrymandering such that a plan is a gerrymander if and only if it produces an above-threshold efficiency gap.

The article proceeds in five parts. Part I relates some necessary background. It describes how partisan cartographers are able to manipulate district lines for political gain, why this manipulation threatens values central to representative democracy, and why this practice has been particularly resistant to precise quantification and judicial oversight.

Part II unpacks the efficiency gap proposal. It presents a mathematically precise explanation of the long-form efficiency gap calculation, expressed in terms of relative wasted votes; the simplified formula, expressed in terms of undeserved seat share; and the associated legal test.

Part III reframes the efficiency gap measure in two mathematically-equivalent but conceptually-distinct ways. First, it shows that by reordering the computation process, we can observe the features of plans that produce zero efficiency gaps. Second, it shows that the efficiency gap can be understood as a competitiveness gap, expressed in terms of turnout and margin of victory, rather than wasted votes or undeserved seat. These two reframings reveal how the efficiency gap measure allows or even encourages mapmakers to draw plans that undermine electoral competitiveness and proportionality between votes earned and seats won.

Part IV generalizes the efficiency gap measure by clarifying and then relaxing the key methodological choices and electoral assumptions on which



it relies. It explains that the definition and weight of surplus votes are methodological choices, not self-defining concepts, and that these choices determine the efficiency gap's relationship to the norms of electoral competitiveness and seats-votes proportionality. It then explores the implications of three further assumptions underlying the efficiency gap: that every race is contested, that voter turnout is equal in each district, and that every ballot is cast for one of two parties. Because the efficiency gap is partially a measure of voter turnout, it fails to register – and may incentivize – voter suppression.

Part V concludes, drawing doctrinal implications from the mathematical and legal analysis preceding it. First, questions of robustness and scope must be addressed when setting the numeric threshold and computing a challenged plan's efficiency gap. Second, given the measure's normatively fraught relationship with competing democratic norms, courts should only use it as an indicative measure and not as the exclusive definition of political gerrymandering.

## **I. BACKGROUND – IN SEARCH OF A STANDARD FOR POLITICAL GERRYMANDERING**

This Part explains how proponents designed the efficiency gap proposal to empower the courts to regulate partisan gerrymandering. Section A explains why gerrymandering is so tempting to mapmakers and so threatening to the multiple democratic values electoral districting implicates. Section B explains why gerrymandering has proven so difficult for the Court to proscribe and the guidance Justice Kennedy has offered on the legal test he seeks.

### *A. The Problem of Political Gerrymandering*

This Section explains how gerrymandering works, why it is so alluring to mapmakers, and so threatening to democracy. The term “gerrymander” is a portmanteau of the surname “Gerry” and the word “salamander.” It was coined in 1812 by a critic of the districting plan for the Massachusetts state senate, who likened its appearance to a salamander, and suggested that Governor Elbridge Gerry was behind it.<sup>16</sup> The term colorfully captures our intuitive sense – and visceral disgust – that manipulation of electoral districts subverts fundamental democratic norms. But the term, like its amphibian namesake, is slippery in the sense that it is imprecise; it does not clarify how gerrymandering operates, nor does it specify which

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<sup>16</sup> Arizona State Legislature v. Arizona Indep. Redistricting Comm'n, 135 S. Ct. 2652, 2658 at n.1 (2015) (describing etymology) (citing E. GRIFFITH, THE RISE AND DEVELOPMENT OF THE GERRYMANDER 16–19 (Arno ed. 1974)); Vieth v. Jubelirer, 541 U.S. 267, 274 (2004) (plurality opinion) (describing etymology) (citing WEBSTER'S NEW INTERNATIONAL DICTIONARY 1052 (2d ed. 1945)).

democratic norms gerrymandering threatens. This Section fleshes out our intuitions and unpacks the concept of Gerry-mandering.

The risk of gerrymandering is an inherent feature of the practice of geographic electoral districting, whereby individual representatives for a multi-member body are selected through separate elections conducted in geographic subunits (called electoral districts) of the entire jurisdiction represented by the entire body. Many countries eschew districting entirely, opting instead for some system of proportional representation, whereby representation of the entire body is distribution according to the support each party earns in a single election conducted over the entire jurisdiction.<sup>17</sup> But from its inception to the present, the American electoral system has relied heavily on geographic electoral districting, increasingly under a single-member simple plurality (SMSP) system, where each electoral district is assigned one seat in the multi-member body, and each district awards its seat to the candidate who earns the most votes in that district's race.<sup>18</sup>

While electoral districting may offer some advantages over proportional representation systems,<sup>19</sup> it has one profound disadvantage: it is vulnerable to manipulation by political cartographers. By carefully adjusting electoral districts lines, the shrewd mapmaker can powerfully influence electoral outcomes, favoring certain candidates, or groups of voters defined on the basis of geography (i.e. rural versus urban), race (white versus black), or party (Republican versus Democrat). To favor one group over another, simply dilute the influence of disfavored voters by assigning them to districts where their votes have less impact: by “*packing*” them into a few districts where their preferred candidates win by overwhelming margins, and/or by “*cracking*” them into many districts so that their preferred candidates lose each one. Aided by powerful computers – and prevailing patterns of residence and voting – the modern mapmaker can pack and crack with exquisite precision, and thereby distort the way political parties translate popular support (votes) into governmental power (seats).<sup>20</sup> With the stroke of a pen

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<sup>17</sup> FAIRVOTE, ELECTORAL SYSTEMS AROUND THE WORLD (2012), [http://www.fairvote.org/research\\_electoralsystems\\_world](http://www.fairvote.org/research_electoralsystems_world) (surveying 35 major democracies, 27 of which use some form of proportional representation).

<sup>18</sup> *What is redistricting?*, All About Redistricting: PROFESSOR JUSTIN LEVITT'S GUIDE TO DRAWING THE ELECTORAL LINES, <http://redistricting.ils.edu/what.php> (last visited Mar. 21, 2017) (“Most of our federal legislators, all of our state legislators, and many of our local legislators in towns and counties are elected from districts. These districts divide states and the people who live there into geographical territories.”).

<sup>19</sup> See Peter H. Schuck, *The Thickest Thicket: Partisan Gerrymandering and Judicial Regulation of Politics*, 87 COLUM. L. REV. 1325 (1987); see also Nathaniel Persily, *In Defense of Foxes Guarding Henhouses: The Case for Judicial Acquiescence to Incumbent-Protecting Gerrymanders*, 116 HARV. L. REV. 649 (2002).

<sup>20</sup> *Vieth*, 541 U.S. at 345–46 (Breyer, J., dissenting) (citing Samuel Issacharoff, *Gerrymandering and Political Cartels*, 116 HARV. L. REV. 593, 624 (2002) [hereinafter Issacharoff, *Political Cartels*]; Pamela S. Karlan, *The Fire Next Time: Reapportionment After the 2000 Census*, 50 STAN. L. REV. 731, 736 (1998); and Richard H. Pildes, *Principled Limitations on Racial and Partisan Redistricting*, 106 YALE L.J. 2505, 2553–2554 (1997)).

(or keyboard), the mapmaker can confer a legislative majority on a party supported by a minority of voters; or a legislative supermajority on a party supported by a slim majority of voters. “Political cartels” can collude, forming bipartisan “non-compete agreements” and carving up an electoral map into “safe” seats for each of the major parties or to benefit incumbents.<sup>21</sup> In this way, electoral districting confers on the mapmaker the power to shape electoral destiny. As one state legislator candidly put it, the practice of gerrymandering turns the process of electoral districting into “the business of rigging elections.”<sup>22</sup> This is why legislatures guard their districting power so jealously,<sup>23</sup> why the districting process is often so partisan and secretive,<sup>24</sup> and why parties expend so many resources drawing and litigating electoral districting plans.<sup>25</sup> Electoral districting entails districting power; such power invites abuse; we call such abuse gerrymandering.

### *B. The Elusiveness of Political Gerrymandering*

To limit abuse of electoral districting, the Court must translate its underlying normative and technical complexity into a manageable legal test discernible from legal text. The Court has adopted legal tests when districting implicates race (racial gerrymandering), or when mapmakers manipulate districts’ population size (malapportionment). But mapmakers can still manipulate district *shape* on the basis of *party*, and in this crucial domain of political gerrymandering, the Court has repeatedly fractured on whether and how to intervene. Its proponents designed the efficiency gap proposal to fill this doctrinal lacuna and empower the Court to regulate political gerrymandering. Before presenting the efficiency gap proposal in the next section, this Section summarizes the problem the Court faces, how it has proceeded in the contexts of race and malapportionment, how it has

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<sup>21</sup> Issacharoff, *Political Cartels*, *supra* note 21, at 621.

<sup>22</sup> *Vieth v. Jubelirer*, 541 U.S. 267, 317 (2004) (Kennedy, J., concurring in the judgment) (quoting Hoeffel, *Six Incumbents Are a Week Away from Easy Election*, WINSTON-SALEM JOURNAL, Jan. 27, 1998, p. B1 (quoting a North Carolina state senator)).

<sup>23</sup> See *Arizona State Legislature v. Arizona Indep. Redistricting Comm’n*, 135 S. Ct. 2652 (2015).

<sup>24</sup> For example, the Wisconsin Assembly plan challenged in *Whitford* was produced with the use of non-disclosure agreements, expedited legislative procedures, a war room with limited access, and consultation exclusively with members of one party. See *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*3-8 (W.D. Wis. Nov. 21, 2016).

<sup>25</sup> See, e.g., Lisa Marshall Manheim, *Redistricting Litigation and the Delegation of Democratic Design*, 93 BOSTON. U. L. REV. 563 (2013); David Daley, *The House the GOP Built: How Republicans Used Soft Money, Big Data, and High-Tech Mapping to Take Control of Congress and Increase Partisanship*, N.Y. Mag. (Apr. 24, 2016). Former President Obama and Attorney General Eric Holder are focusing on redistricting reform through a newly formed organization called the National Democratic Redistricting Committee. Edward-Isaac Dove, *Obama, Holder to lead post-Trump redistricting campaign*, POLITICO (Oct. 17, 2016).

repeatedly fractured on political gerrymandering, and the guidance Justice Kennedy has provided on the legal test he seeks.

First, consider the problem electoral districting presents to the Court. For as long as mapmakers have gerrymandered, critics have called on the United State Supreme Court to end (or at least curb) the practice. Yet, as Justice Kennedy observes, excessive judicial regulation of electoral districting “would commit federal and state courts to unprecedented intervention in the American political process,” and without “rules to limit and confine judicial intervention...intervening courts—even when proceeding with best intentions—would risk assuming political, not legal, responsibility for a process that often produces ill will and distrust.”<sup>26</sup>

The question is not just whether political gerrymandering is a constitutional problem, but whether the Court can address it. In the language of federal courts jurisprudence, the question is whether gerrymandering constitutes a justiciable legal claim the courts can adjudicate or a nonjusticiable political question the courts cannot address, which turns on whether or not the Court can identify a judicially discernible and manageable standard to channel and limit judicial intervention. Thus gerrymandering presents two distinct but related questions: justiciability (is there a standard?) and identification (what is it?).

These questions implicate both the relationship between electoral districting practices and constitutionally-significant representative norms and the proper role of the federal judiciary in regulating electoral districting practices pursuant to these norms. Both are democratic problems of profound constitutional significance on which the Constitution provides limited explicit guidance. Although political gerrymanders undoubtedly implicate constitutional values,<sup>27</sup> the Constitution’s text offers limited procedural guidance on congressional and state legislative elections.<sup>28</sup> And the precise

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<sup>26</sup> *Vieth v. Jubelirer*, 541 U.S. 267, 306–07 (2004) (Kennedy, J., concurring in the judgment).

<sup>27</sup> *Vieth*, 541 U.S. at 293 (plurality opinion) (“[A]n excessive injection of politics is unlawful.”); *Arizona State Legislature v. Arizona Indep. Redistricting Comm’n*, 135 S. Ct. 2652, 2658 (2015) (“[T]his Court has recognized [that gerrymanders] are incompatible with democratic principles.”) (quoting *Vieth*, 541 U.S. at 292 (plurality opinion)) (internal quotations omitted). The Constitution provides that Representatives are to be “chosen . . . by the People,” U.S. CONST. ART. I, SEC. 2, guarantees each State “a Republican Form of Government,” U.S. CONST. ART. IV, SEC. 4; protects freedoms of expression and association, U.S. CONST. AMEND. I; enshrines due process and equal protection, U.S. CONST. AMEND. XIV, SEC. 1; and prohibits race-based electoral discrimination, U.S. CONST. AMEND. XV, SEC. 1.

<sup>28</sup> Article I vests the federal legislative power in a Congress composed of two multimember legislative bodies: a House, apportioned on a population basis and popularly elected, U.S. CONST. ART. I, SEC. 2, CL. 1 & 3; and a Senate, apportioned on the basis of equal state suffrage, U.S. CONST. ART. I, SEC. 1, CL. 3 (two Senators per State); U.S. CONST. ART. V. (constraining amendment of Senate apportionment), with each State’s Senators originally “chosen by the Legislature thereof,” U.S. CONST. ART. I, SEC. 1, CL. 3, and now “elected by the people thereof.” U.S. CONST. AMEND. XVII (ratified in 1913). But the Elections Clause

scope of the federal judicial power is also a question without an explicit textual answer.<sup>29</sup> In this sense, both action and inaction by the Court on political gerrymandering claims present real but ineffable constitutional risks.

When electoral districting implicates race, the Court and Congress have imposed two principal legal constraints. First, pursuant to the Reconstruction Amendments, the Court evaluates a districting plan under strict scrutiny whenever considerations of race predominate, thereby curtailing intentional race-based cracking or packing.<sup>30</sup> Second, the prohibition on racial vote dilution codified in Section 2 of the Voting Rights Act, as amended in 1982 and construed by the Court, constrains the ability of a mapmaker to crack a sufficiently numerous, politically cohesive and geographically compact group of minority voters in the presence of racially polarized voting.<sup>31</sup>

Outside the context of race, the Court has been more hesitant to intervene. For decades, the Court dismissed malapportionment challenges as nonjusticiable political questions, heeding Justice Powell’s admonition not to enter “the political thicket,”<sup>32</sup> before reversing course, adopting the one-person-one-vote principle, and thereby launching the reapportionment

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does not mandate how to conduct congressional elections; instead, it gives the choice to the State and Congress. U.S. CONST. ART. I, SEC. 4, CL. 1. And while the Constitution assumes that each State has at least one popularly elected legislative body, *see, e.g.*, U.S. CONST. ART. I, SEC. 2, CL. 1; U.S. CONST. AMEND. XVII, it says nothing explicit about how to conduct state legislative elections, implicitly leaving that choice to each State as well.

<sup>29</sup> Article III, Section 2 provides that the “judicial Power shall extend” to an enumerated set of “Cases” and “Controversies”, the first of which is “all Cases, in Law and Equity, arising under [federal law].” While “it is emphatically the province and duty to say what the law is,” *Marbury v. Madison*, 5 U.S. 137, 177 (1803), some “subjects are political [and so] can never be examinable by the courts.” *Marbury v. Madison*, 5 U.S. 137, 166 (1803). The political-question doctrine, like all justiciability doctrines partially discerned from the text of Article III, “relate in part, and in different though overlapping ways, to an idea, which is more than an intuition but less than a rigorous and explicit theory, about the constitutional and prudential limits to the powers of an unelected, unrepresentative judiciary in our kind of government.” *Elk Grove Unified Sch. Dist. v. Newdow*, 542 U.S. 1, 11 (2004) (quoting *Vander Jagt v. O’Neill*, 699 F.2d 1166, 1178–1179 (D.C. Cir. 1983) (Bork, J., concurring)).

<sup>30</sup> *Gomillion v. Lightfoot*, 364 U.S. 339 (1960); *Shaw v. Reno*, 509 U.S. 630 (1993); *Bethune-Hill v. Virginia State Board of Elections*, 137 S. Ct. 788 (2017); *Cooper v. Harris*, 137 S. Ct. 1455 (2017).

<sup>31</sup> See Pub. L. No. 89-110, 79 Stat 437, 437, codified as amended at 52 USC §10301; *Thornburg v. Gingles*, 478 U.S. 30 (1986); *Grove v. Emison*, 507 U.S. 25 (1993); *Johnson v. De Grandy*, 512 U.S. 997 (1994); *Bartlett v. Strickland*, 556 U.S. 1 (2009). Section 5 of the Voting Rights Act imposed one additional constraint: a covered jurisdiction could only implement a new districting plan after persuading the Department of Justice or a federal court that the plan would not have a racially retrogressive effect. But since the Court struck down the coverage formula, *see Shelby County v. Holder*, 133 S. Ct. 2612 (2013), and Congress has yet to adopt a new one, this third constraint on racial gerrymandering is presently inoperative.

<sup>32</sup> *Colegrove v. Green*, 328 U.S. 549, 556 (1946).

revolution.<sup>33</sup> In subsequent cases, the Court translated this constitutional principle into a reasonably stable and coherent doctrinal framework based on a simple numeric measure: the relevant constitutional ideal is population equality; maximum population deviation (essentially, a *population gap*) measures the extent of departure from this constitutional ideal; a numeric threshold of 10% is set for state legislative districts; an above-threshold population gap triggers a presumption of invalidity, which a state can only overcome by adequately justifying the population gap as the consequence of legitimate and consistently applied districting criteria; and a below-threshold population gap triggers a presumption of validity, which a challenger can overcome by showing the population gap is a consequence of an illegitimate purpose, like a desire to obtain partisan advantage.<sup>34</sup>

Thus, the Court has constrained race-based manipulation of district shape and political manipulation of district size – meaningfully limiting mapmakers’ packing and cracking ability. But manipulation of district *shape* based on *party* still flies under the judicial radar.<sup>35</sup> And, perversely, by requiring endless redistricting, the one-person-one-vote standard gave mapmakers new opportunities to manipulate electoral district boundaries,<sup>36</sup> making electoral districting a moving target resistant to judicial oversight. The lingering unresolved challenge for the Court, then, is how to develop a discernible and manageable legal test for political gerrymandering.

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<sup>33</sup> K ELBRIDGE GERRY’S SALAMANDER: THE ELECTORAL CONSEQUENCES OF THE REAPPORTIONMENT REVOLUTION (2002); GORDON BAKER, THE REAPPORTIONMENT REVOLUTION (1966); Gordon E. Baker, *The Unfinished Reapportionment Revolution*, in POLITICAL GERRYMANDERING AND THE COURTS 11, 11-25 (1990).

<sup>34</sup> Reynolds v. Sims, 377 U.S. 533 (1964) (applying the “one person one vote” principle to state legislative elections under the Equal Protection Clause of the Fourteenth Amendment); Wesberry v. Sanders, 376 U.S. 1 (1964) (applying the “one person one vote” principle to congressional elections under Article I, Section 2, Clause 1); Avery v. Midland County, 390 U.S. 474 (1968) (applying the “one person one vote” principle to general purpose local government elections under the Equal Protection Clause of the Fourteenth Amendment); Brown v. Thomson, 462 U.S. 835 (1983) (establishing threshold of 10% maximum population deviation for state legislative plans).

<sup>35</sup> Without a clear federal legal test for political gerrymandering, plaintiffs have an incentive to attack political gerrymanders as racial gerrymanders, and defendants have an incentive to justify partisan plans as efforts to comply with the Voting Rights Act. In the context of “conjoined polarization” – the “more consistent alignment of race, party, and ideology since” passage of the original Voting Rights Act, Bruce E. Cain & Emily R. Zhang, *Blurred Lines: Conjoined Polarization and Voting Rights*, 77 OHIO ST. L.J. 867, 869 (2016), race and party are easy for litigants to conflate and hard for courts to distinguish. See Richard L. Hasen, *Race or Party, Party as Race, or Party All the Time: Three Uneasy Approaches to Conjoined Polarization in Redistricting and Voting Cases*, \_\_\_\_ WM. & MARY L. REV. \_\_\_\_ (forthcoming 2018). Cooper v. Harris, 137 S. Ct. 1455 (2017).

<sup>36</sup> Pamela S. Karlan, *John Hart Ely and the Problem of Gerrymandering: The Lion in Winter*, 114 YALE L.J. 1329, 1339 (2005) (“[W]e have moved from entrenchment through inaction to a perhaps even more pathological phenomenon of entrenchment through nonstop action.”).

In *Bandemer*, three Justices argued that political gerrymandering was a political question,<sup>37</sup> but six Justices insisted it was justiciable under the Equal Protection Clause<sup>38</sup> and agreed that plaintiffs must demonstrate both discriminatory intent and discriminatory effect.<sup>39</sup> Yet those six Justices disagreed among themselves on the correct legal test for discriminatory effect.<sup>40</sup> For the next 18 years, the lower courts applied Justice White’s standard – and rejected at the motions stage every political gerrymandering claim they considered.<sup>41</sup>

In *Vieth*, the four conservative Justices thought political gerrymandering was a political question.<sup>42</sup> The four liberal Justices insisted it was justiciable, but offered three different legal tests.<sup>43</sup> Justice Kennedy rejected each standard proposed,<sup>44</sup> but insisted that political gerrymandering

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<sup>37</sup> *Davis v. Bandemer*, 478 U.S. 109, 144 (1986) (O’Connor, J., concurring in the judgment) (joined by Chief Justice Burger and Justice Rehnquist); *id.* at 143 (Burger, C.J., concurring in the judgment).

<sup>38</sup> *Id.* at 113 (plurality opinion) (joined by Justices Brennan, Marshall, and Blackmun); *id.* at 161 (Powell, J., concurring in part and dissenting in part) (joined by Justice Stevens).

<sup>39</sup> *Id.* at 127 (plurality opinion); *id.* at 161 (Powell, J., concurring in part and dissenting in part).

<sup>40</sup> Justice White, writing for a four-Justice plurality, proposed a stringent but vague “consistent degradation” test under which a departure from seats-votes proportionality would be insufficient to establish discriminatory intent. *Id.* at 132 (plurality opinion) (joined by Justices Brennan, Marshall, and Blackmun) (“the mere lack of proportional representation will not be sufficient . . . Rather, unconstitutional discrimination occurs only when the [challenged plan] will consistently degrade a voter’s or a group of voters’ influence on the political process as a whole.”). Justice Powell, joined by Justice Stevens, proposed a standard that would have been easier for courts to apply and plaintiffs to meet. *Id.* at 173 & n.13 (Powell, J., concurring in part and dissenting in part) (proposing consideration of multiple factors including “the shapes of voting districts . . . adherence to established political subdivision boundaries . . . legislative procedures . . . population disparities and . . . disproportionate election results”).

<sup>41</sup> *Vieth v. Jubelirer*, 541 U.S. 267, 279-80 (2004) (plurality opinion) (“[I]n *all* of the cases we are aware of involving that most common form of political gerrymandering, relief was denied.”) (emphasis in original); *id.* at 280 n. 6 (citing cases).

<sup>42</sup> *Id.* at 281 (plurality opinion) (authored by Scalia, J. and joined by Rehnquist, C.J., O’Connor, J., and Thomas, J.).

<sup>43</sup> Justice Stevens suggested a “predominant motivation standard” based on the *Shaw v. Reno* cause of action for racial gerrymandering. *Id.* at 341 (Stevens, J., dissenting). Justice Souter proposed a burden-shifting framework modelled on Title VII doctrine with a five-factor *prima facie* case. *Id.* at 347 (Souter, J., joined by Ginsberg, J., dissenting). Justice Breyer proposed a test based on “unjustified [partisan] entrenchment,” whereby a party with a minority of vote share achieves a majority of seat share through “partisan manipulation.” *Id.* at 360 (Breyer, J., dissenting). The *Vieth* plaintiffs proposed to demonstrate discriminatory effect by showing that the challenged plan “systematically ‘pack[s]’ and ‘crack[s]’ the rival party’s voters [and thereby threatens to] thwart the plaintiffs’ ability to translate a majority of votes into a majority of seats.” *Id.* at 286–87 (plurality opinion) (citing Brief for Appellants at 20).

<sup>44</sup> *Id.* at 308 (Kennedy, J., concurring in the judgment).

may be justiciable.<sup>45</sup> Justice Kennedy emphasized that judicial intervention required “clear, manageable, and politically neutral standards for measuring the particular burden a given partisan classification imposes on representational rights,”<sup>46</sup> and suggested the First Amendment may offer a better textual basis than the Equal Protection Clause for such standards.<sup>47</sup>

In *LULAC*, the Court fractured along similar lines.<sup>48</sup> Justice Scalia, joined by Justice Thomas, continued to insist that political gerrymandering was a political question.<sup>49</sup> The four liberal justices continued to favor justiciability and suggest alternative legal tests.<sup>50</sup> Justice Kennedy again rejected each proffered standard, while expressing hope an adequate standard may yet materialize.<sup>51</sup> But this time, political scientists Gary King, Bernard Grofman, Andrew Gelman, and Jonathan Katz proposed a new “partisan symmetry” standard based explicitly on the seats-votes framework.<sup>52</sup> Their standard “require[d] that the electoral system treat similarly-situated parties equally, so that each receives the same fraction of legislative seats for a particular vote percentage as the other party would receive if it had received the same percentage [of the vote].”<sup>53</sup>

The liberal Justices expressed interest in the partisan symmetry concept.<sup>54</sup> But Justice Kennedy identified three concerns with this proposal:

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<sup>45</sup> *Id.* at 306 (Kennedy, J., concurring in the judgment).

<sup>46</sup> *Id.* at 307-08 (Kennedy, J., concurring in the judgment).

<sup>47</sup> *Id.* at 314 (Kennedy, J., concurring in the judgment) (“The First Amendment may be the more relevant constitutional provision in future cases that allege unconstitutional partisan gerrymandering.”); *id.* at 315 (Kennedy, J., concurring in the judgment) (“Where it is alleged that a gerrymander had the purpose and effect of imposing burdens on a disfavored party and its voters, the First Amendment may offer a sounder and more prudential basis for intervention than does the Equal Protection Clause.”).

<sup>48</sup> The case fractured the Court in particularly severe and complex fashion because it presented both an unsuccessful claim of political gerrymandering and a successful claim of racial vote dilution of Section 2 of the Voting Rights Act.

<sup>49</sup> *LULAC v. Perry*, 548 U.S. 399, 511 (2006) (Scalia, J., concurring in the judgment in part and dissenting in part) (joined by Thomas, J.).

<sup>50</sup> *Id.* at 447 (Stevens, J., dissenting in part); *id.* at 483 (Souter, J., concurring in part and dissenting in part) (joined by Ginsberg J.); *id.* at 491 (Breyer, J., concurring in part and dissenting in part).

<sup>51</sup> *Id.* at 408. Chief Justice Roberts, joined by Justice Alito, agreed with Justice Kennedy’s conclusion that plaintiffs failed to provide an adequate standard, but declined to weigh in on the question of justiciability. *Id.* at 492–93 (Roberts, C.J., concurring in part, concurring in the judgment in part, and dissenting in part) (joined by Alito, J.)

<sup>52</sup> Amicus Brief in *Jackson v. Perry* Submitted on Behalf of Neither Party by Gary King, Bernard Grofman, Andrew Gelman, and Jonathan Katz in the U.S. Supreme Court (No. 05-276).

<sup>53</sup> *Id.* at 4-5.

<sup>54</sup> *LULAC*, 548 U.S. at 466 (Stevens, J., dissenting) (“the symmetry standard, a measure social scientists use to assess partisan bias...is undoubtedly ‘a reliable standard...’); *id.* at 483–84 (Souter, J.) (declining to “rule out the utility” of the symmetry standard, noting that “[i]nterest in exploring this notion is evident” and suggesting “further attention could be devoted to [its] administrability”).



(1) it involved “conjecture”;<sup>55</sup> (2) it relied on a counterfactual “hypothetical” rather than a directly observed election;<sup>56</sup> and (3) it provided no guidance on how much departure from the ideal is “too much.”<sup>57</sup>

## II. UNPACKING THE EFFICIENCY GAP

It is against this backdrop that the proponents of the efficiency gap measure entered the scene. The efficiency gap is designed to do for shape manipulation what the population gap does for size manipulation. It is a simple number that measures deviation from a constitutional ideal as the basis of a stable constitutional framework to constrain gerrymandering.

The “efficiency gap” proposal was presented in the academic literature through two related articles, a political science article published in 2014 by Eric McGhee,<sup>58</sup> (hereinafter, the “2014 article”) and a 2015 law review article co-authored by Eric McGhee and Nicholas Stephanopoulos.<sup>59</sup> (hereinafter, the “2014 article”) I refer to the authors as the academic proponents or proponents. The 2014 article introduced the numeric measure and demonstrated its key technical properties, while the 2015 article developed the measure into a proposed legal standard. Plaintiffs then adopted an efficiency gap approach in litigation challenging Wisconsin’s state assembly plan (the *Whitford* litigation) and North Carolina’s congressional plan (the *Rucho* litigation). In 2017, McGhee published an article that addressed previously unexamined technical and conceptual aspects of the efficiency gap measure, offering a more refined and generalized conceptualization (hereinafter, the “2017 article”).

Through these two articles, the academic proponents present a powerful case for the efficiency gap measure, and associated legal test.

The efficiency gap measure makes a great first impression. It is a new and improved measure of partisan symmetry designed to “capture[] in a single tidy number, all of the packing and cracking decisions that go into a district plan...”<sup>60</sup> and thereby distill “the essence of what critics have in mind when they refer to partisan gerrymandering.”<sup>61</sup> The concept is powerful and intuitive. But underlying its apparent conceptual simplicity lies mathematical complexity. To engage this complexity, we must translate the concept from

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<sup>55</sup> *Id.* at 420 (opinion of Kennedy, J.) (“The existence or degree of asymmetry may in large part depend on conjecture about where possible vote-switchers will reside.”).

<sup>56</sup> *Id.* (opinion of Kennedy, J.) (“[W]e are wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs.”).

<sup>57</sup> *Id.* (opinion of Kennedy, J.) (“[T]he counterfactual plaintiff would face the same problem as the present, actual appellants: providing a standard for deciding how much partisan dominance is too much.”).

<sup>58</sup> McGhee, *supra* note 9.

<sup>59</sup> Stephanopoulos & McGhee, *supra* note 7.

<sup>60</sup> *Id.* at 831.

<sup>61</sup> *Id.* at 852.

words to math. A precise, technical framing reveals insights that a linguistic formulation obscures, clarifying the measure’s underlying premises and its performance relative to other democratic norms such as competitiveness and strict proportionality. This section provides a technical account of the measure, first in terms of the long-form computation and second in terms of the simplified formula that reframes the measure in terms of undeserved seat share. It then explains the proponents’ associated legal test, which is designed to address Justice Kennedy’s concerns with prior proposals.

A. *The Proponents’ Definitional Framing: Relative Wasted Votes*

The proponents define the efficiency gap using a long-form formula based on the concept of wasted votes. This section explains the long-form formula and the methodological choices and electoral assumptions upon which it relies.

The academic proponents define “[a] gerrymander [as] . . . a district plan that results in one party wasting many more votes than its adversary.”<sup>62</sup> “Wasted votes include both ‘lost’ votes (those cast for a losing candidate) and ‘surplus’ votes (those cast for a winning candidate but in excess of what she needed to prevail).”<sup>63</sup> This definition reflects the insight that, through “packing” and “cracking,” a partisan mapmaker can win seats by distributing the disfavored party’s votes in inefficient ways: packing leads to wasting many surplus votes, while cracking leads to wasting many lost votes. This definition takes a view of gerrymandering centered on the democratic norm of symmetric partisan efficiency.

The key objective of the efficiency gap is to capture “all of a plan’s cracking and packing choices into a single number.”<sup>64</sup> The efficiency gap is a measure of the relative “wasted” or “inefficient” votes of each of two main parties across a districting plan.<sup>65</sup> The efficiency gap measures the existence and extent of a gerrymander by “indicat[ing] the magnitude of the divergence between the parties’ respective wasted votes.”<sup>66</sup> To calculate the efficiency gap, “[e]ach party’s wasted votes are totaled, one sum is subtracted from the other, and then . . . this difference is divided by the total number of votes cast.”<sup>67</sup> In this calculation, “lost” and “surplus” (or “excess”) votes are weighted the same. The logic here is that “[i]n . . . plurality-rule, single-member district (SMD) elections . . . ‘inefficient’ votes are those that do not directly contribute to victory. Thus, any vote for a losing candidate is wasted

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<sup>62</sup> *Id.* at 849-50.

<sup>63</sup> *Id.* at 851.

<sup>64</sup> *Id.* at 849-50.

<sup>65</sup> The 2014 article first introduced the concept, referring to it as “relative wasted votes,” McGhee, *supra* note 9, at 68; and it was later developed into the concept of the “efficiency gap.” Stephanopoulos & McGhee, *supra* note 7, at 834 & n.12.

<sup>66</sup> Stephanopoulos & McGhee, *supra* note 7, at 849-50.

<sup>67</sup> *Id.* at 851-52.

by definition, but so too is any vote beyond the 50 percent threshold needed (in a two-candidate race) to win a seat.”<sup>68</sup>

This intuitive linguistic formulation relies on three related methodological choices that will prove critical in my analysis: (1) the “two-party” assumption; (2) the definition and weight of surplus votes; and (3) the aggregation method from wasted votes by party and district to the efficiency gap for the entire plan. I consider each of these choices in turn.

### 1. The Two-Party Assumption

First, the proponents make the electoral assumption that “there are only two parties.”<sup>69</sup> The efficiency gap measure is by definition a bilateral comparison – it takes two parties and compares their relative efficiency by calculating the difference in their respective wasted vote totals. I use the term “focal parties” to refer to the two parties that are the focus of the efficiency gap measure’s bilateral comparison, and the term “peripheral candidates” to refer to candidates unaffiliated with the two focal parties. I call the two focal parties Party X and Party Y.<sup>70</sup> When proponents assume “there are only two parties,” they not only concentrate on the two focal parties subject to the bilateral comparison, but also ignore any ballot cast for a peripheral candidate. Because proponents emphasize partisan fairness between the two major political parties,<sup>71</sup> they make the simplifying assumption that every district race is a contest between one Party X candidate and one Party Y candidate. This “two-party” assumption actually consists of three related assumptions: (1) in each district, no ballots are cast for peripheral candidates; (2) in each district, no more than one candidate runs from each focal party; and (3) each district race is contested, so that a candidate runs from each focal party.<sup>72</sup>

The “two-party assumption” significantly simplifies the analysis. In each district, the analyst counts the number of ballots cast for the Party X candidate ( $V_{xi}$ ) and the number of ballots cast for the Party Y candidate ( $V_{yi}$ ).

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<sup>68</sup> *Id.* at 850-51.

<sup>69</sup> *Id.* at 853.

<sup>70</sup> The efficiency gap is a signed measure. Its absolute value indicates the extent of the gerrymander, while its sign indicates which party the gerrymander favors. I define all relevant concepts so that a positive gap favors Party X and a negative gap favors Party Y. Obviously, the two primary parties of interest are the major political parties. Whenever I discuss the efficiency gap between Republicans and Democrats, I adopt the convention that Party X connotes Republicans and Party Y connotes Democrats.

<sup>71</sup> Stephanopoulos & McGhee, *supra* note 7, at 831 (characterizing the efficiency gap as “a new measure of *partisan* symmetry” designed to capture “the idea that a plan should treat the *major parties* symmetrically.”) (emphasis supplied).

<sup>72</sup> See McGhee, *supra* note 9, at 68; Stephanopoulos & McGhee, *supra* note 7, at 853 & n.114. In this section, I use the “two-party” assumption to analyze the efficiency gap on its own terms. In Part IV, I relax the assumptions.

The candidate who receives more votes wins, and the district's total voter turnout ( $T_i$ ) is simply the sum of the candidate's respective vote totals.

$$T_i = V_{yi} + V_{xi}$$

Just as every ballot is cast either for Party X or for Party Y, every district race is won either by (the candidate for) Party X or (the candidate for) Party Y. Thus, the set of districts ( $D$ ) can be split into the set of X-won districts ( $D_x$ ) and the set of Y-won districts ( $D_y$ ).

But this simplification is achieved at the expense of ignoring significant real-world electoral phenomena, specifically third-party, independent and write-in candidates; electoral competition between candidates of the same party, which occurs in states like California, Washington, and Louisiana that use a "top-two" or "jungle" primary system; and uncontested races, which often occur, particularly in state legislative elections, and particularly in less competitive states with less competitive plans.<sup>73</sup>

## 2. Weight and Definition of Surplus Votes

Second, proponents make two related methodological choices when defining a "wasted vote": (1) they define a surplus vote using a threshold of half of actual voter turnout; and (2) they equally weight lost and surplus votes. Specifically, proponents adopt the following mathematical definition of the number of votes wasted ( $W_{pi}$ ) by a given party ( $p$ ) in a given district ( $i$ ) depending on whether the party wins ( $i \in D_p$ ) or loses ( $i \in D'_p$ ).

$$W_{pi} = \begin{cases} E_{pi} = V_{pi} - \frac{T_i}{2} & D_p \\ L_{pi} = V_{pi} & D'_p \end{cases}$$

When a party loses ( $i \in D'_p$ ), all its votes are lost and thus wasted. This means the number of wasted votes for the party ( $W_{pi}$ ) is the number of lost votes for the party ( $L_{pi}$ ), which is the number of ballots cast for the party ( $V_{pi}$ ). When a party wins ( $i \in D_p$ ), only those votes beyond the threshold needed to win are excess<sup>74</sup> and thus wasted. The number of wasted votes for the party ( $W_{pi}$ ) is the number of excess votes for the party ( $E_{pi}$ ), which is the

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<sup>73</sup> Part III.C analyzes the application of the efficiency gap to plans with uncontested races.

<sup>74</sup> I use the term "excess" instead of "surplus," so that I can use the letter  $E$  to denote the concept, rather than the letter  $S$ , which could be confused with "seat" or "share."

difference between the number of ballots cast for the party ( $V_{pi}$ ) and the number of votes needed to win, defined as half of actual voter turnout ( $T_i$ ).<sup>75</sup>

This is one way to define and weight surplus votes. But it is not the only way, nor is it the most intuitive way. Part III.A relaxes the definition and weight of surplus votes, and derives a more generalized formula that quantifies the precise impact of these methodological choices on the efficiency gap measure. This generalized formula demonstrates the critical role these methodological choices play in calibrating the measure's relationship between competing norms of electoral competitiveness and seats-votes proportionality.

### 3. Aggregation from Wasted Votes to Efficiency Gap

Finally, the proponents aggregate values by *party* and *district* to produce a single number for the entire *plan*. For each party, the number of wasted votes over the entire plan is simply the sum over all districts of wasted votes in each district.

$$W_p = \sum_D W_{pi}$$

The plan's efficiency gap ( $\Delta W_{xy}$ ) is "the difference between the parties' respective wasted votes, divided by the total number of votes cast in the election."<sup>76</sup>

$$\Delta W_{xy} = \frac{W_y - W_x}{V_y + V_x}$$

This approach compares the parties' relative wasted vote totals: it is zero when each party wastes the same raw number of votes. Alternatively, one could define the efficiency gap as the parties' relative waste vote shares so that the gap is zero when each party wastes the same proportion of the votes it earns.

$$\Delta W_{xy} = \frac{W_y}{V_y} - \frac{W_x}{V_x}$$

Part III.B considers this alternative approach to scaling the efficiency gap.

Proponents approach involves a two-step aggregation process: first sum wasted votes over districts; second compute the difference.<sup>77</sup> Part II.B switches the order of aggregation by defining a district-level wasted vote disparity and then expressing a plan's efficiency gap as the weighted average

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<sup>75</sup> Stephanopoulos & McGhee, *supra* note 7, at 851.

<sup>76</sup> *Id.*

<sup>77</sup> *Id.* at 851-52. "Each party's wasted votes are totaled, one sum is subtracted from the other, and then...this difference is divided by the total number of votes cast."

of district-level disparities. Using this new approach, I clarify the relationship between the measure's district-level and plan-level performance.

*B. The Proponents' Alternative Framing: Undeserved Seat Share*

After defining the efficiency gap measure with the long-form formula, the proponents derive a simplified formula that recasts the measure from one of relative wasted votes to one of undeserved seat share. This section presents the essential technical steps.

First, the proponents make one final assumption: the equal turnout assumption, or the assumption that “each district has exactly the same number of voters.”<sup>78</sup> Mathematically, this means for every district ( $i$ ) in the plan ( $D$ ), district-level turnout ( $T_i$ ) is equal to average turnout level ( $T^*$ ).

$$T_i = T^* = \frac{\sum_D T_i}{d} \quad \forall i \in D$$

Second, they use this assumption to reduce the long-form equation to the following simplified formula:<sup>79</sup>

$$\Delta W = S^* - 2V^*$$

This simplified formula expresses the efficiency gap ( $\Delta W$ ) as a simple function of the statewide vote margin ( $S^*$ ) and the statewide vote margin ( $V^*$ ) for Party X.

Finally, they use the simplified formula to reframe the efficiency gap as a measure of undeserved seat share. According to this formula, in an ideal world without partisan gerrymandering, each party's seat margin ( $S^*$ ) would be twice its vote margin ( $V^*$ ).

$$\Delta W = 0 \leftrightarrow S^* = 2V^*$$

The efficiency gap, as calculated by the simplified formula, can be understood graphically as the vertical distance between the observed vote-seat combination and the “ideal” one associated with this vote-seat curve. On this basis, proponents present the gap as a measure of the “undeserved seat

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<sup>78</sup> McGhee, *supra* note 9, at 79 (Appendix B) (deriving the simplified formula) (first line of derivation explicitly assumes that “each district has exactly the same number of voters” so that “proportions can be substituted for raw votes in all of the formulas [and so] the total vote in each district becomes equal to 1.0,” and the sum of district vote totals “is simply the total number of districts in the electoral system.”); Stephanopoulos & McGhee, *supra* note 7, at 853 (citing McGhee's derivation of the simplified formula in Appendix B of his 2014 article). Section IV.C discusses and relaxes this assumption.

<sup>79</sup> See McGhee, *supra* note 9, at 79 (Appendix B) (deriving the simplified formula); *infra* note 118.

share” attributable to partisan gerrymandering rather than the party’s popularity.

Like the seats-votes curve for strict proportionality, this one is a straight line. But this one has a slope of *two* instead of *one*. This means that, according to the efficiency gap, the ideal seats-votes relationship is one of double-responsiveness and double-proportionality. As the proponents explain, “[T]he gap offers what scholars to date have been unable to supply: a normative guide as to how large [the seat] bonus should be. To produce partisan fairness—in the sense of equal wasted votes for each party—the bonus should be a precisely twofold increase in seat share for a given increase in vote share.”<sup>80</sup>

The seats-votes curve associated with the simplified formula runs through the point where each party equally splits votes and seats (0.5,0.5). At this one point, both strict proportionality and equal wasted votes are achieved. When one party earns more than half the votes, there is tension between the goal of equal wasted votes and the goal of strict proportionality.

### C. *The Legal Test*

The 2015 article proposed a legal test for political gerrymandering that incorporates the efficiency gap measure. If the plan’s efficiency gap exceeds a numeric threshold, and sensitivity analysis suggests that the plan will continue to produce an above-threshold gap in future elections, it is presumptively invalid.<sup>81</sup> This presumption can only be overcome if the plan’s partisan effect can be justified or explained as the product of legitimate redistricting criteria consistently applied to the jurisdiction’s underlying political geography.<sup>82</sup>

The academic proponents originally proposed a numeric threshold of 8% based on their analysis of historical practice – gaps above 8% represent outliers relative to the distribution of gaps produced by modern electoral maps.<sup>83</sup> The *Whitford* plaintiffs’ expert, Professor Simon Jackman, proposed a numeric threshold of 7% based on durability – gaps above 7% tend to persist for the life of an electoral map.<sup>84</sup>

The proposed efficiency gap measure and associated legal test was crafted to appeal to Justice Kennedy. It is explicitly framed as an effort to improve upon the partisan bias proposal offered by amici and considered by the Court in *LULAC*.<sup>85</sup> It relies on an intuitive and constitutionally discernible concept of symmetric partisan treatment viewed favorably by five justices in

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<sup>80</sup> Stephanopoulos & McGhee, *supra* note 7, at 854.

<sup>81</sup> *Id.* at 885, 891.

<sup>82</sup> *Id.* at 884.

<sup>83</sup> *Id.* at 884.

<sup>84</sup> *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*15 (W.D. Wis. Nov. 21, 2016). [cite report] [provide details]

<sup>85</sup> Stephanopoulos & McGhee, *supra* note 7, at 895.

*LULAC*, while addressing the inadequacies Justice Kennedy identified with the symmetry measure proposed by the *LULAC* amici.<sup>86</sup> The measure of partisan symmetry proposed by the *LULAC* amici necessarily relied on assumptions to compare party performance in a hypothetical counterfactual.<sup>87</sup> The efficiency gap compares party performance directly observed in actual election results without necessarily relying on inferential techniques.<sup>88</sup> And the proposed efficiency gap test answers the question of how much advantage is too much<sup>89</sup> with a numeric threshold of presumptive validity set on the bases of historical practice and durability. An above-threshold efficiency gap is a directly-observed indication that the electoral map favors one party in a way that is likely to persist for the life of the map and that departs from historical practice. Finally, the measure is distinct from a requirement of strict proportionality, a standard the Court has already rejected. Unlike strict proportionality, this double-proportionality measure permits some seat bonus, but limits the size of this bonus. The limit aligns with electoral reality – political scientists have consistently found a 2-to-1 votes-to-seats ratio in state legislative and congressional elections.<sup>90</sup> And the limit is simple: seats-votes responsiveness can exceed one but not two.

### III. PROPORTIONALITY AND COMPETITIVENESS

#### A. *Relevant Democratic Norms*

Jurists, scholars, politicians, media, reformers, and ordinary citizens agree that gerrymandering poses a profound threat to democratic values.<sup>91</sup>

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<sup>86</sup> *Id.* at 895-99.

<sup>87</sup> *LULAC v. Perry*, 548 U.S. 399, 420 (2006) (Kennedy, J.) (“The existence or degree of asymmetry may in large part depend on conjecture about where possible vote-switchers will reside.”); *id.* at 420 (Kennedy, J.) (“wary of adopting a constitutional standard that invalidates a map based on unfair results that would occur in a hypothetical state of affairs.”).

<sup>88</sup> Stephanopoulos & McGhee, *supra* note 7, at 857. However, inferential techniques are used for sensitivity analysis and in the case of uncontested districts.

<sup>89</sup> *LULAC*, 548 U.S. at 420 (Kennedy, J.) (“plaintiff . . . [must] provid[e] a standard for deciding how much partisan dominance is too much.”).

<sup>90</sup> Nicholas M. Goedert, *Redistricting, Risk, and Representation: How Five State Gerrymanders Weathered the Tides of the 2000s*, 13 ELECTION L. J. 8 (2014); Edward R. Tufte, *The Relationship between Seats and Votes in Two-Party Systems*, 67 AMERICAN POLITICAL SCIENCE REVIEW, 542 (1973). The *Whitford* majority emphasized that both parties stipulated that the simplified formula’s “implied 2-to-1 votes-to-seats relationship reflects the ‘observed average seat/votes curve in historical U.S. congressional and legislative elections.’” *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*54 (W.D. Wis. Nov. 21, 2016).

<sup>91</sup> *Arizona Indep. Redistricting Comm’n*, 135 S. Ct. at 2658 (“inconsistent with democratic principles”); Robert Draper, *The League of Dangerous Mapmakers*, ATLANTIC (Sept. 19, 2012, 8:56 PM), <http://www.theatlantic.com/magazine/archive/2012/10/the-league-of/309084/> (“the most insidious practice in American politics.”); DAVID DALEY, *RATF\*\*KED: THE TRUE STORY BEHIND THE SECRET PLAN TO STEAL AMERICA’S*



But there is disagreement on the right way to draw electoral districts, and thus disagreement on precisely how to define and measure gerrymandering.<sup>92</sup> Gerrymandering, as a word and a concept, is slippery because electoral districting implicates, and gerrymandering threatens, multiple democratic values – including electoral competition, voter participation, and partisan fairness. Democratic health, like human health, is an irreducibly multidimensional concept.<sup>93</sup> To fully grasp gerrymandering – a democratic disease – we must engage the multidimensionality of democratic structure. But just as doctors may disagree on the most salient components of health at stake in a treatment decision, legal scholars “are divided as to what [is] the most important structural consideration” that “capture[s] what is truly at stake” in electoral districting.<sup>94</sup> “Two approaches to redistricting have dominated the academic debate over the last generation: the partisan fairness approach, advocating that district plans treat the major parties symmetrically, and the competitiveness approach, advising that districts be made as competitive as is feasible.”<sup>95</sup> The competitiveness approach emphasizes electoral competition between candidates and parties.<sup>96</sup> Whereas competitive races promote responsiveness and accountability,<sup>97</sup> safe districts shift the action from the general election to the primary, pushing legislators to ideological extremes, promoting polarization and gridlock,<sup>98</sup> and reducing

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DEMOCRACY (2016); Jack M. Balkin, *The Last Days of Disco: Why the American Political System Is Dysfunctional*, 94 B.U. L. REV. 1159, 1165 (2014) (including “exclusively single-member districts,” “first-past-the-post election rules,” and “[p]olitical gerrymandering” in a list of “features of our current system that make it dysfunctional”).

<sup>92</sup> *Vieth v. Jubelirer*, 541 U.S. 267, 307 (2004) (Kennedy, J. concurring in the judgment) (“No substantive definition of fairness in districting seems to command general assent.”); Krasno, *supra* note 16, at 1 (“Partisan gerrymandering shares both of the characteristics of pornography that Potter Stewart famously wrestled with in his concurring opinion in *Jacobellis*: it is difficult to measure objectively and (therefore) a matter of subjective opinion.”).

<sup>93</sup> Heather K. Gerken, *Lost in the Political Thicket: The Court, Election Law, and the Doctrinal Interregnum*, 153 U. PA. L. REV. 503, 508 (2004) (“It is hard to figure out what is “fair” or “equal” in districting without speaking in structural terms. Any such conclusion would require a theory of representation, an idea about how a healthy democracy is supposed to function.”).

<sup>94</sup> Nicholas O. Stephanopoulos, *Elections and Alignment*, 114 COLUM. L. REV. 283, 296-97 (2014) [hereinafter Stephanopoulos, *Alignment*].

<sup>95</sup> Nicholas O. Stephanopoulos, *The Consequences of Consequentialist Criteria*, 3 UC IRVINE L. REV. 669, 673 (2013) [hereinafter Stephanopoulos, *Consequentialist Criteria*].

<sup>96</sup> See, e.g., Issacharoff, *Political Cartels*, *supra* note 21; Samuel Issacharoff & Richard H. Pildes, *Politics as Markets: Partisan Lockups of the Democratic Process*, 50 STAN. L. REV. 643 (1998); Stephanopoulos, *Alignment*, *supra* note 30, at 365 n.62 (2014) (citing scholarship).

<sup>97</sup> Stephanopoulos, *Consequentialist Criteria*, *supra* note 31, at 676–77 (citing scholarship).

<sup>98</sup> Lillian V. Smith, *Recreating the “Ritual Carving”*: *Why Congress Should Fund Independent Redistricting Commissions and End Partisan Gerrymandering*, 80 BROOK. L. REV. 1641, 1644 (2015) (gerrymandering “reduc[es] the competitiveness of elections, contribut[es] to the systemic entrenchment of partisan interests, and perpetuat[es]

the responsiveness of legislators to the general electorate.<sup>99</sup> The proliferation of safe districts<sup>100</sup> also discourages high-quality challengers, reduces party mobilization, and depresses voter participation,<sup>101</sup> giving incumbents an advantage unrelated to their prior performance or present popularity. An analyst can quantify a districting plan's electoral competitiveness by computing the average margin of victory or share of competitive seats it produces in a real or hypothetical election.<sup>102</sup>

The partisan fairness approach focuses on the relationship between the number of votes a party earns and the number of seats it wins.<sup>103</sup> Equal treatment of the parties is normatively appealing in a way that closely tracks constitutional principles. It coheres with “the First Amendment interest of not burdening or penalizing citizens because of their participation in the electoral process, their voting history, their association with a political party, or their expression of political views.”<sup>104</sup>

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congressional deadlock.”). Recent Congresses have been among the least productive in history, voting dozens of times to overturn Obamacare but struggling to avoid government shutdowns and defaults on the national debt; launching investigations into alleged misconduct, but refusing to hold hearings on the President's proposed budget or Supreme Court nominee. When legislators fear primary challengers but take general elections for granted, the predictable result is increased partisanship and gridlock.

<sup>99</sup> Samuel Issacharoff & Pamela S. Karlan, *Where to Draw the Line?: Judicial Review of Political Gerrymanders*, 153 U. PA. L. REV. 541, 574 (2004) (“The perverse consequence of the incumbent gerrymander is that it skews the distribution politically by driving the center out of elected office at the legislative level.”); Josh Chafetz, *The Phenomenology of Gridlock*, 88 NOTRE DAME L. REV. 2065, 2086 (2013) (considering the possibility that “the combination of partisan primaries and bipartisan gerrymandering are resulting in a legislature that cannot be said to be broadly responsive to the American people.”).

<sup>100</sup> By one count, the number of swing congressional districts plunged from 103 in 1992 to 35 in 2012. Nate Silver, *As Swing Districts Dwindle, Can a Divided House Stand?*, N.Y. TIMES (Dec. 27, 2012), <http://fivethirtyeight.blogs.nytimes.com/2012/12/27/as-swing-districts-dwindle-can-a-divided-house-stand/>. See also John Nichols, *Why Redistricting Threatens Democracy*, THE NATION (Feb. 20, 2012), <https://www.thenation.com/article/why-redistricting-threatens-democracy/> (redistricting “explains why the vast majority of races are not competitive.”).

<sup>101</sup> E.g., *LULAC v. Perry*, 548 U.S. 399, 470–71 n.10 (2006) (Stevens, J., dissenting in part); Richard H. Pildes, *The Constitution and Political Competition*, 30 NOVA L. REV. 253, 260 (2006) (“[I]t is well documented that competitive elections encourage the appearance of strong challengers to incumbents and increase voter turnout and party mobilization.”). Note that some scholars and jurists emphasize voter participation directly as a primary democratic value. See Stephanopoulos, *Alignment*, *supra* note 30, at 297 (describing Justice Breyer, Chris Elmendorf, and Spencer Overton's views on the primacy of voter participation).

<sup>102</sup> See Stephanopoulos, *Consequentialist Criteria*, *supra* note 31, at 671.

<sup>103</sup> See, e.g., Bernard Grofman & Gary King, *The Future of Partisan Symmetry as a Judicial Test for Partisan Gerrymandering After LULAC v. Perry*, 6 ELECTION L.J. 2, 6 (2007).

<sup>104</sup> *Vieth v. Jubelirer*, 541 U.S. 267, 314 (2004) (Kennedy, J., concurring in the judgment).

Political scientists formally analyze some of the key democratic norms at stake by drawing a “seats-votes curve” to illustrate how a party translates popular support (vote share) into governing power (seat share).<sup>105</sup> An electoral outcome corresponds to a single point; a curve represents a range of outcomes. Seats-votes *proportionality* captures the *absolute* relationship between vote share and seat share – the ratio between total vote share and total seat share. Seats-votes *responsiveness* captures the *marginal* relationship – the ratio between an incremental change in vote share and the corresponding incremental change in seat share.

If a seats-votes curve is a straight line, *proportionality* and *responsiveness* are equal. For example, a straight line through the origin with a 45-degree angle corresponds to strict proportionality, where a party’s seat share is simply its vote share. Some argue for an ideal of strict proportionality between a party’s vote share and its seat share, and on this basis propose that we replace our districting-based electoral system with one explicitly based on proportional representation.<sup>106</sup>

In the real world, the actual seats-votes curves estimated by political scientists are not straight lines, but S-shaped curves that exhibit lower responsiveness (flatter slopes) when one party enjoys a large majority and higher responsiveness (steeper slopes) when the electorate is more evenly split between the two parties.

The result is a “seat bonus,”<sup>107</sup> whereby the majority translates a positive vote margin into an even larger seat margin. Some argue this is normatively desirable, because it incentivizes robust campaigning;<sup>108</sup> others argue it is normatively undesirable because it departs from strict proportionality.<sup>109</sup>

The democratic norms of competitiveness and partisan fairness are distinct but intimately related. Seats-votes responsiveness is linked to electoral competitiveness; the more districts with close elections, the more a small change in vote share produces a large change in seat share.<sup>110</sup> This suggests intimate technical relationships – and normative trade-offs – between norms of proportionality, responsiveness, competitiveness, and participation.

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<sup>105</sup> A descriptive seat-votes curve estimates the relationship that actually exists in the real world. A prescriptive seats-votes curve indicates the ideal relationship that ought to exist in a healthy, well-functioning democracy.

<sup>106</sup> DOUGLAS J. AMY, *REAL CHOICES / NEW VOICES: HOW PROPORTIONAL REPRESENTATION ELECTIONS COULD REVITALIZE AMERICAN DEMOCRACY* (2d ed. 2002).

<sup>107</sup> Adam Cox, *Partisan Fairness and Redistricting Politics*, 79 N.Y.U. L. REV. 751, 765 (2004); Mitchell N. Berman, *Managing Gerrymandering*, 83 TEX. L. REV. 781, 854 (2005).

<sup>108</sup> E.g., Peter H. Shuck, *The Thickest Thicket: Partisan Gerrymandering and Judicial Regulation of Politics*, 87 COLUM. L. REV. 1325, 1350–51 (1987).

<sup>109</sup> E.g., AMY, *supra* note 42.

<sup>110</sup> Stephanopoulos, *Consequentialist Criteria*, *supra* note 31, at 678 (“[E]lectoral responsiveness indicates . . . how competitive individual districts are . . .”).

Thus, gerrymandering is slippery in both a normative and technical sense. For this reason, it has repeatedly eluded the Court's grasp.

*B. Zero-Disparity Districts and Zero-Gap Plans*

This section examines the relationship between the efficiency gap, competitiveness, and seats-votes proportionality. To do this, I employ a three-step analytic technique. First, I define the wasted vote disparity at the level of an individual district and express the efficiency gap as an average of the district-level wasted vote disparities. Second, I define a simple zero-gap plan as one composed entirely of zero-disparity districts. Third, I show that an analyst can convert any zero-gap plan to and from a simple zero-gap by performing the appropriate series of voter swaps, where two districts exchange two voters, one supporting each party.

Using this analytic technique, I clarify the relationship between the efficiency gap, competitiveness, and seats-votes proportionality. At the level of an individual district, the wasted vote disparity is a function of competitiveness. In a zero-disparity district, one party earns three quarters of ballots cast and thus prevails by a margin of victory equal to half of turnout. A more competitive win produces a wasted vote disparity in favor of the winner; a less competitive win favors the loser. And there is a discontinuity when the seat flips parties, such that a small change in votes produces a huge change in the wasted vote disparity. A plan's efficiency gap is the difference between the seat margin and twice the vote margin. But it is also the seat-share-weighted difference between the average competitiveness of X-won districts and the average competitiveness of Y-won districts. And the sensitivity of a plan's gap to changes in vote share is a function of how many districts have small victory margins. Since a party can win a zero-disparity district with three quarters of ballots cast, a party can win every seat in a zero-gap plan with 75% vote share. And a party can win 75% seat share with 59% vote share and a below-threshold efficiency gap.

The particular analytic technique I employ here is not needed to derive these substantive results. It is a straightforward exercise to derive them using the standard aggregation method. Moreover, I recognize that the efficiency gap measure was designed, is intended, and should only be used at the level of an entire plan, not an individual district. But examining and then aggregating over district-level disparities is a useful analytic technique to explore the plan-level operation of the efficiency gap measure. It simplifies the math. And it helps me understand in a deeper and more intuitive way not just how the measure operates, but why. I thus offer this technique in the hope it will prove similarly help to the reader.

1. [A District's Wasted Vote Disparity Is a Discontinuous Linear Function of Its Margin of Victory](#)

I begin by defining the district-level wasted vote disparity ( $\Delta w_{xyi}$ ) between Party X and Party Y in district ( $i$ ) as the parties' relative wasted votes in that district, expressed as a proportion of district-level voter turnout.

$$\Delta w_{xyi} = \frac{W_{yi} - W_{xi}}{T_i}$$

A district's disparity is a function of its competitiveness. Recall that, under the two-party assumption, every ballot is cast either for Party Y or for Party X, and the sum of the party's respective vote totals is the district's total voter turnout:  $T_i = V_{yi} + V_{xi}$ . Let  $V_{i1}$  and  $V_{i2}$  respectively denote the number of ballots cast for the first and second place candidates. The difference is the victory margin ( $M_i$ ) – a primary measure of electoral competitiveness.<sup>111</sup>

$$M_i = V_{i1} - V_{i2} = \begin{cases} V_{xi} - V_{yi} & D_x \\ V_{yi} - V_{xi} & D_y \end{cases}$$

The proportional margin of victory ( $m_i$ ) is the district-level margin of victory ( $M_i$ ) expressed as a proportion of district-level voter turnout ( $T_i$ ):  $m_i = \frac{M_i}{T_i}$ . Rearranging, we can express  $V_{i1}$  and  $V_{i2}$  in terms of victory margin ( $M_i$ ) and turnout ( $T_i$ ).<sup>112</sup>

$$V_{i1} = \frac{T_i + M_i}{2} = \frac{T_i}{2} (1 + m_i)$$

$$V_{i2} = \frac{T_i - M_i}{2} = \frac{T_i}{2} (1 - m_i)$$

Let  $m_p = \frac{\sum_p m_i}{d_p}$  denote the average proportional victory margin in p-won districts and let  $m = \frac{\sum_p m_i}{d}$  denote the average proportional victory margin over all districts. Note that the seat-share-weighted sum of  $m_x$  and  $m_y$  is  $m$  and the seat-share-weighted difference of  $m_x$  and  $m_y$  is twice the vote margin ( $V$ ).

$$m = S_x m_x + S_y m_y$$

$$2V = S_x m_x - S_y m_y$$

<sup>111</sup> See Stephanopoulos, *Consequentialist Criteria*, *supra* note 31, at 678. The sign of  $M_i$  indicates which party wins, while its absolute value indicates the winner's margin of victory expressed in whole number of ballots.  $M_i$  is positive when Party X wins ( $D_x$ ) and negative when Party Y wins ( $D_y$ ).

<sup>112</sup>  $\frac{T_i}{2} (1 + m_i) = \frac{T_i + M_i}{2} = \frac{(V_{i1} + V_{i2}) + (V_{i1} - V_{i2})}{2} = V_{i1}$   
 $\frac{T_i}{2} (1 - m_i) = \frac{T_i - M_i}{2} = \frac{(V_{i1} + V_{i2}) - (V_{i1} - V_{i2})}{2} = V_{i2}$

This means we can express  $m_x$  and  $m_y$  in terms of  $m$ ,  $V$  and  $S$ .<sup>113</sup>

$$m_x = \frac{m + 2V}{1 + 2S}$$

$$m_y = \frac{m - 2V}{1 - 2S}$$

At the level of an individual district, the vote disparity ( $\Delta w_{xyi}$ ) is a discontinuous linear function of the victory margin ( $m_i$ ).

$$\Delta w_{xyi} = \begin{cases} m_i - \frac{1}{2} & D_x \\ \frac{1}{2} - m_i & D_y \end{cases}$$

The disparity ( $\Delta w_{xyi}$ ) is zero when the victor prevails by half of turnout<sup>114</sup> - regardless of which party wins. With this victory margin, the winner and the loser earn votes at a 3:1 ratio, so three quarters of ballots are cast for the winner, one quarter for the loser. Thus, the winner's minimizing vote share ( $v^*$ ) is 75%.

Define the set of zero-disparity or minimizing districts ( $\Pi_i^0$ ) as the set of districts that produce a wasted vote disparity of zero.

$$\Pi_i^0 = P_i \mid \Delta w_{xyi}(P_i) = 0$$

Let  $P_i^0$  denote a minimizing district,  $P_i^{x,0}$  an x-won minimizing district, and  $P_i^{y,0}$  a y-won minimizing district.

$$P_i^{x,0} = T_i\left(\frac{3}{4}, \frac{1}{4}\right), P_i^{y,0} = T_i\left(\frac{1}{4}, \frac{3}{4}\right)$$

<sup>113</sup> Proof:

$$\begin{aligned} \frac{m + 2V}{1 + 2S} &= \frac{(S_x m_x + S_y m_y) + (S_x m_x - S_y m_y)}{2\left(\frac{1}{2} + S\right)} = \frac{2S_x m_x}{2S_x} = m_x \\ \frac{m - 2V}{1 - 2S} &= \frac{(S_x m_x + S_y m_y) - (S_x m_x - S_y m_y)}{2\left(\frac{1}{2} - S\right)} = \frac{2S_y m_y}{2S_y} = m_y \end{aligned}$$

<sup>114</sup> This makes intuitive sense. The proponents note that precisely half of all votes are wasted. Stephanopoulos & McGhee, *supra* note 7 at 851 and n. 107. Thus, equal wasted votes occur when each party wastes one quarter of ballots cast. Mira Bernstein & Moon Duchin, *A Formula Goes to Court: Partisan Gerrymandering and the Efficiency Gap*, Physics and Society (forthcoming 2017), p.4, available at <https://arxiv.org/pdf/1705.10812.pdf>.

Define the minimizing victory margin ( $m^*$ ) as one half, and the competitiveness score ( $c_i$ ) as the difference between the minimizing ( $m^*$ ) and the actual ( $m_i$ ) victory margin. Then the wasted vote disparity is:

$$\Delta w_{xyi} = \begin{cases} c_i & D_x \\ -c_i & D_y \end{cases}, c_i = m^* - m_i, m^* = 0.5$$

$$\Delta w_{xyi} = c_i = 0 \leftrightarrow m_i = m^* = 0.5$$

A district's disparity is simply its competitiveness score, the difference between the minimizing and the actual victory margin, with a sign convention such that a relatively competitive district favors the winning party while a relatively uncompetitive district favors the losing party.<sup>115</sup>

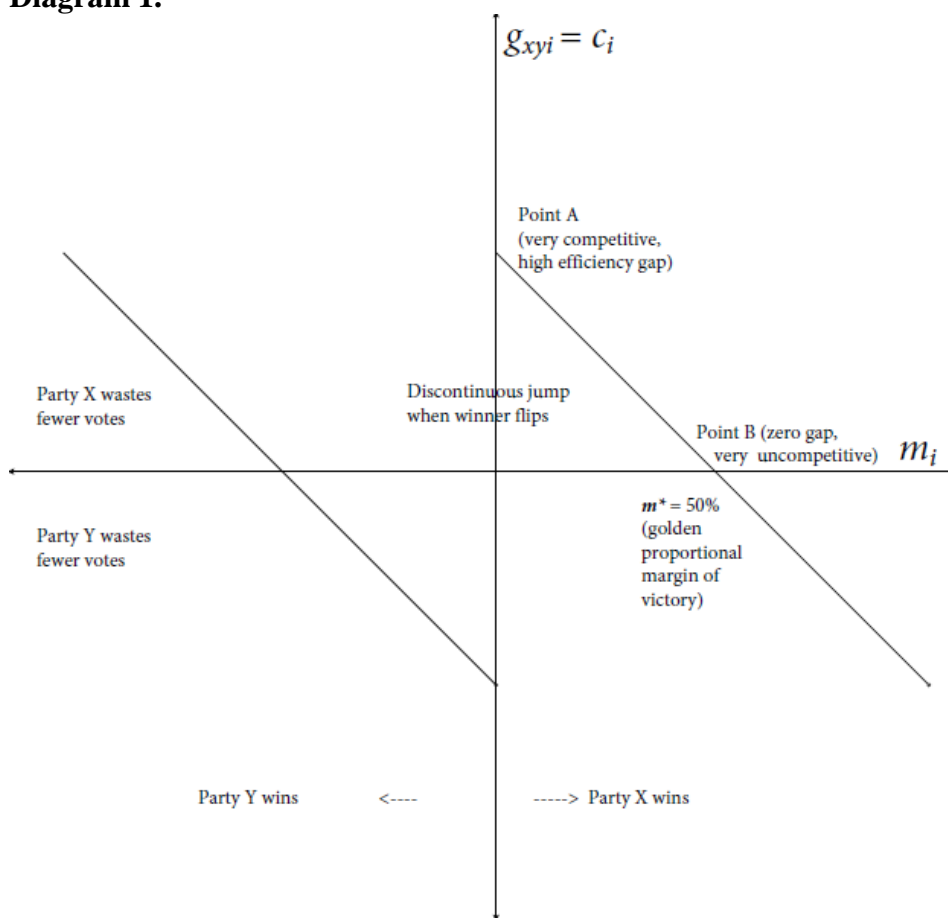
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<sup>115</sup> When  $m_i$  exceeds  $m^*$ ,  $c_i$  is negative because the district is less competitive than the minimizing level. This favors the losing party, because surplus votes exceed lost votes. When  $m^*$  exceeds  $m_i$ ,  $c_i$  is positive because the district is more competitive than the minimizing level. This favors the winning party because lost votes exceed surplus votes. The district level disparity is positive (favors Party X) when X wins a relatively competitive district or Y wins a relatively uncompetitive district; the disparity is negative (favors party Y) when X wins a relatively uncompetitive district or Y wins a relatively competitive district.

Diagram 1 illustrates the relationship between a district's disparity and the vote share for party X. The diagram looks like a "double back slash" – two downward sloping lines with a discontinuous jump at the 50% mark when the seat flips from one party to the other.

As the following sections flesh out, much of the measure's operation at the plan-level can be divined from this district-level relationship. Since a party can win a single seat with a zero-plan disparity and 75% of the vote, a party can win all the seats with a zero-plan gap and 75% of the votes. Since a district's disparity is its competitiveness score, the efficiency gap is the seat-share-weighted difference in average competitiveness. And since a district's disparity jumps discontinuously when a seat flips, the sensitivity of a plan's efficiency gap to vote swings is a function of how many of its districts are competitive.

**Diagram 1.**





## 2. The Competitiveness Gap

A plan's efficiency gap ( $\Delta W_{xy}(P)$ ) is the weighted average of district-level wasted vote disparities, where each district's wasted vote disparity ( $\Delta w_{xyi}(P_i)$ ) is weighted by its turnout ( $T_i$ ).<sup>116</sup>

$$\Delta W_{xy} = \frac{\sum_D T_i \Delta w_{xyi}}{\sum_D T_i}$$

Under the equal turnout assumption, the plan's efficiency gap is simply the unweighted average district disparity.

$$\Delta W_{xy} = \frac{\sum_D \Delta w_{xyi}}{d}$$

Let  $c_p = \frac{\sum_p c m_i}{d_p}$  denote the average competitiveness score in p-won districts and let  $c = \frac{\sum_p c_i}{d}$  denote the average competitiveness score over all districts. Note that average competitiveness is the difference between the minimizing and the average victory margin;  $c = m^* - m$ ,  $c_p = m^* - m_p$ ; and  $c$  is the seat-share-weighted sum of  $c_x$  and  $c_y$ .  $c = S_x c_x + S_y c_y$ . Define the competitiveness gap ( $\Delta C_{xy}$ ) as the seat-share-weighted difference of  $c_x$  and  $c_y$ .

$$\Delta C_{xy} = S_x c_x - S_y c_y$$

Under equal voter turnout, the efficiency gap is the competitiveness gap.

$$\Delta W_{xy} = \frac{\sum_D \Delta w_{xyi}}{d} = \frac{\sum_{D_x} c_i + \sum_{D_y} -c_i}{d} = \frac{d_x c_x - d_y c_y}{d} = S_x c_x - S_y c_y = \Delta C_{xy}$$

And the competitiveness gap reduces to the simplified seats-votes formula.

$$\begin{aligned} \Delta C_{xy} &= S_x(m^* - m_x) - S_y(m^* - m_y) \\ &= \frac{1}{2}(S_x - S_y) - (S_x m_x - S_y m_y) = S^* - 2V^* \end{aligned}$$

Thus, under equal turnout, one can frame the measure as relative wasted votes, undeserved seat share, or differential average competitiveness.

$$\Delta W_{xy} = S^* - 2V^* = \Delta C_{xy}$$

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<sup>116</sup> Appendix 2.

Note that a zero efficiency gap does not necessarily entail equal average competitiveness in X-won and Y-won districts. This equality obtains only in the special case where each party wins half the seats. In the more general case, the relative competitiveness of X-won and Y-won districts needed to achieve a zero efficiency gap will depend on the relative seat shares. Formally, let  $\mathbf{c} = \frac{c_x}{c_y}$  denote the competitiveness ratio, and  $\mathbf{s} = \frac{s_x}{s_y}$  denote the seat ratio. Then the sign of the efficiency gap ( $\Delta W$ ) depends on which is bigger the seat ratio ( $\mathbf{s}$ ) or the competitiveness ratio ( $\mathbf{c}$ ).

$$\begin{cases} \Delta W > 0 & \mathbf{c} > \mathbf{s} \\ \Delta W = 0 & \mathbf{c} = \mathbf{s} \\ \Delta W < 0 & \mathbf{c} < \mathbf{s} \end{cases}$$

When the competitiveness ratio exceeds the seat ratio, the competitiveness gap is positive; when the seat ratio exceeds the competitiveness ratio, the competitiveness gap is negative; when the competitiveness ratio equals the seat ratio, the competitiveness gap is zero.

### 3. The Simple Plan and Voter Swaps

Define the set of zero-gap or minimizing plans ( $\Pi^0$ ) as the set of plans that produce an efficiency gap of zero.

$$\Pi^0 = \mathbf{P} \mid \Delta W_{xy}(\mathbf{P}) = 0$$

One way to construct a minimizing plan is to create a plan composed entirely of minimizing districts with equal voter turnout.

$$\mathbf{P}^* = \bigcup \mathbf{P}_i^0$$

I call such a plan a simple minimizing plan. By design, it maintains both the equal voter turnout assumption and a zero plan-level efficiency gap.<sup>117</sup> It has a zero competitiveness gap, because each district has a zero competitiveness score. Since the plan must assign each voter to one district, and each district is won by half of turnout ( $\mathbf{m}_i = \mathbf{m}^* = \frac{1}{2}$ ), the seat margin must be double

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<sup>117</sup> Since both parties waste the same number of votes in each district, the parties must waste the same number of votes overall.  $\Delta W_{xy}(\mathbf{P}^*) = \frac{\sum_D \Delta w_{xyi}(\mathbf{P}_i^0)}{d} = \frac{\sum_D 0}{d} = 0$ .

the vote margin.<sup>118</sup> This offers yet another way to understand the double proportionality and double responsiveness that emerges from the proponents' approach: it is the seats-votes relationship exhibited by a simple minimizing plan composed exclusively of minimizing districts. When each party earns half the votes, the simple minimizing plan accords each party half the seats. When one party earns 75% of the votes, the simple minimizing plan accords that party all the seats, because each district is won by that party 75-25.

The simple minimizing plan is not the only minimizing plan. More generally, a plan can achieve a zero plan-level gap even if it exhibits non-zero district-level wasted vote disparities, so long as these disparities average out so that both parties waste the same number of votes overall.  $\Delta W_{xy} = \frac{\sum_D \Delta w_{xyi}}{d} = 0$ . But each minimizing plan can be converted to (and from) a minimizing plan by performing the appropriate series of voter swaps, whereby two districts swap two voters – one Party X supporter for one Party Y supporter - without altering any district election outcome.<sup>119</sup> By design, a swap preserves equal voter turnout<sup>120</sup> and statewide vote share.<sup>121</sup> Since no swap alters a district's election outcome, the series preserves seat share. Since the series preserves vote share, seat share, and equal turnout, it must preserve the efficiency gap.

As Table 1 illustrates, a swap may be classified by the winning party in  $P_i$  and  $P_j$ . In a type I swap (x wins both),  $P_i$  becomes less competitive but  $P_j$  becomes more competitive, so the average x-won district competitiveness score ( $c_x$ ) remains the same. In a type IV swap (y wins both),  $P_i$  becomes more competitive but  $P_j$  becomes less competitive, so the average y-won district competitiveness score ( $c_x$ ) remains the same. In a type

<sup>118</sup> In the  $d_x$  x-won districts, party x earns  $\frac{T^*}{2}$  more votes, and in the  $d_y$  y-won districts party x earns  $\frac{T^*}{2}$  fewer votes. Thus:  $2V^* = \frac{V_x - V_y}{v} = \frac{T^*(\sum_{D_x} m^* + \sum_{D_y} -m^*)}{dT^*} = \frac{\frac{1}{2}(d_x - d_y)}{d} = S^*$

<sup>119</sup> Formally, if we start with an equal-turnout plan ( $P = \{V_{xk}, V_{yk}\}$  in ET), and take two districts  $P_i$  and  $P_j$ , a voter swap  $(i, j)(P)$  produces a new plan  $P' = \{V'_{xk}, V'_{yk}\}$  with the following properties:

$$V'_{xk} = \begin{cases} V_{xk} + 1 & k = i \\ V_{xk} - 1 & k = j \\ V_{xk} & k \neq i, j \end{cases} \quad V'_{yk} = \begin{cases} V_{yk} - 1 & k = i \\ V_{yk} + 1 & k = j \\ V_{yk} & k \neq i, j \end{cases}$$

In this swap, district  $P_i$  swaps an x supporter for a y supporter, district  $P_j$  swaps a y supporter for an x supporter, and all other districts remain unchanged.

$$^{120} T'_k = V'_{xk} + V'_{yk} = \begin{cases} V_{xk} + 1 + V_{yk} - 1 & k = i \\ V_{xk} - 1 + V_{yk} + 1 & k = j \\ V_{xk} + V_{yk} & k \neq i, j \end{cases} = V_{xk} + V_{yk} = T_k = T^*$$

$$^{121} V'_x = \sum_k V'_{xk} = V_{xi} + 1 + V_{xj} - 1 + \sum_{k \neq i, j} V_{xk} = \sum_k V_{xk} = V_x$$

$$V'_y = \sum_k V'_{yk} = V_{yi} - 1 + V_{yj} + 1 + \sum_{k \neq i, j} V_{yk} = \sum_k V_{yk} = V_y$$

II swap (x wins  $P_i$ , y wins  $P_j$ ), both  $P_i$  and  $P_j$  become less competitive, so the competitiveness score remains the same. In a type III swap (y wins  $P_i$ , x wins  $P_j$ ), both  $P_i$  and  $P_j$  become less competitive, so the competitiveness score remains the same. In all cases, the competitiveness gap stays the same; but a type II swap decreases overall competitiveness, a type III swap increases overall competitiveness, and type I and IV swaps maintain overall competitiveness.

**Table 1**

Type	$P_i$	$P_j$
I	X	X
II	X	Y
III	Y	X
IV	Y	Y

Starting with a simple minimizing plan, such a swap produces a plan that is still minimizing but no longer simple. Thus, the appropriate series of outcome-preserving swaps can convert any minimizing plan to or from a simple minimizing plan.

#### 4. Unacknowledged Measure Convergence

This reframing suggests that scholars and jurists may be referring to the efficiency gap measure (or something quite like it) without realizing it. For example, Sam Wang recently proposed three tests for partisan gerrymandering, including the following “lopsided outcomes test”:

Test 2 (the lopsided outcomes test): Compare the difference between the share of Democratic votes in the districts that Democrats win, and the share of Republican votes in the districts that Republicans win. This test works because in a partisan gerrymander, the targeted party wins lopsided victories in a small number of districts, while the gerrymandering party’s wins are engineered to be relatively narrow. To compare the winning vote shares for the two parties, I use a grouped t-test, an extremely common statistical test.<sup>122</sup>

Just like the efficiency gap (EG), this “lopsided outcomes test” (LOT) measures differential average competitiveness. There are three differences, but only one is substantive. First, EG focuses on the competitiveness score ( $c_i$ ), whereas LOT focuses on the victor’s vote share ( $v_i^*$ ) – the two are

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<sup>122</sup> Samuel S.-H. Wang, *Three Tests for Practical Evaluation of Partisan Gerrymandering*, 69 STAN. L. REV. 1263, 1304, 1306 (2016).

linearly related -  $v_i^* = \frac{3-2c_i}{4}$ .<sup>123</sup> Second, LOT uses a statistical test to estimate the likelihood that the average competitiveness in X-won districts is different from the average competitiveness in Y-won districts, whereas EG simply computes the difference. The third difference is substantive: LOT measures unweighted differential average competitiveness, assuming that the competitiveness ratio should be one, whereas EG measures seat-share-weighted differential average competitiveness, assuming the competitiveness ratio should be equal to the seat ratio. As the seat ratio approaches one, this last difference drops out. But when the seat ratio departs significantly from one, the two tests will diverge. When one party wins most seats, EG – but not LOT – permits a “competitiveness bonus.”

Similarly, the *Whitford* majority separated its discriminatory effect analysis into two parts. First, the majority presented evidence of discriminatory effect other than the efficiency gap. Only after concluding that “the evidence we have just described certainly makes a firm case on the question of discriminatory effect” did the majority proceed to discuss how “that evidence is further bolstered by the plaintiffs’ use of the “efficiency gap.”<sup>124</sup> But as part of its initial discussion, the majority focused on the competitiveness gap, noting that Democratic-won districts were far less competitive on average than Republican-won districts – that Democrats were packed into “safe” districts.<sup>125</sup> The majority did not realize that in talking about differential competitiveness, it was actually talking about the efficiency gap.

## 5. Unpacking Versus De-Cracking

The efficiency gap is a function of differential competitiveness, not overall competitiveness. To achieve a zero efficiency gap, the competitiveness ratio must equal the seat ratio, but overall competitiveness can take on any value. A zero-gap plan can be highly competitive or highly uncompetitive, so long as it is not *differentially* competitive. The mapmaker can design minimizing plans with districts as competitive or uncompetitive as she pleases while still achieving a zero efficiency gap. She can even make some districts more competitive than others, so long as the relative competitiveness of the average X-won district and the average Y-won district is proportional to the relative number of seats won by parties X and Y. What she cannot do (if she wants a zero or low efficiency gap) is systematically vary competitiveness by party so that the competitiveness ratio departs from

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$$^{123} v_i^* = \frac{V_{i1}}{T_i} = \frac{T_i + M_i}{2T_i} = \frac{1}{2} + \frac{m_i}{2} = \frac{1}{2} + \frac{\frac{1}{2} - c_i}{2} = \frac{1}{2} + \frac{1}{4} - \frac{c_i}{2} = \frac{3-2c_i}{4}$$

<sup>124</sup> *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*80 (W.D. Wis. Nov. 21, 2016).

<sup>125</sup> *Id.* at \*74-75.

the seat ratio. For example, the mapmaker cannot achieve a zero or low efficiency gap by drawing a plan that awards each party half the seats but with Party X generally winning competitive races and Party Y generally winning by landslides. That competitiveness gap would entail an efficiency gap in favor of Party X. It would systematically pack and crack Party Y supporters, which would produce an efficiency gap (Party Y would waste more votes), or equivalently, a competitiveness gap (X-won districts would be more competitive), or equivalently, undeserved seat share relative to the ideal seats-votes ratio of two.

To eliminate (or limit) this undeserved seat share, the mapmaker must flip one or more seats from the favored party to the disfavored party. But just as there are two fundamental gerrymandering strategies – packing and cracking – so there are two analogous strategies to flip the requisite seat(s) and thereby eliminate (or reduce) a large efficiency/competitiveness gap: *unpacking* and *de-cracking*. Unpacking flips the requisite seats by transferring supporters of the disfavored party from relatively uncompetitive districts won by the disfavored party. This unpacking strategy makes the districts won by the disfavored party, and thus the plan over all, more competitive. De-cracking flips the requisite seats by transferring supporters of the disfavored party from relatively competitive districts won by the favored party. This de-cracking strategy makes the districts won by the favored party, and thus the plan over all, less competitive. Table 2 demonstrates these two strategies.

Table 2

DISTRICT	ORIGINAL PLAN		UNPACKING PLAN		DECRACKING PLAN	
	X	Y	X	Y	X	Y
1	700	300	700	300	800	200
2	700	300	700	300	800	200
3	700	300	700	300	700	300
4	540	460	440	560	440	560
5	540	460	440	560	440	560
6	540	460	540	460	540	460
7	540	460	540	460	540	460
8	540	460	540	460	540	460
9	350	650	450	550	350	650
10	350	650	450	550	350	650

In both the unpacking plan and the decracking plan, districts 4 and 5 swap enough x voters for y voters to flip those seats from x-won to y-won, thereby producing a seat margin twice the vote margin and a zero efficiency gap. But the unpacking plan swaps voters out of y-won districts 9 and 10, making those districts and the plan overall more competitive. In contrast, the decracking

plan swaps voters out of x-won districts 1 and 2, making those districts and the plan overall less competitive.

This illustrates how both competitive and uncompetitive plans can produce a zero efficiency gap. But while a plan of any competitiveness level can achieve a zero (or low) gap at one level of vote share, only a plan with a specified level of competitiveness can maintain a zero (or low) gap over a range of vote share. This is so because the sensitivity of the efficiency gap is a function of overall competitiveness.

#### 6. The Sensitivity of the Efficiency Gap to Input Variation Is a Function of Overall Competitiveness

Ignoring turnout effects, the efficiency gap ( $\Delta W$ ) is a function of two variables – the statewide vote margin ( $V^*$ ) and the statewide seat margin ( $S^*$ ):  $\Delta W = S^* - 2V^*$ . But the statewide seat margin ( $S^*$ ) is itself a function of the statewide vote margin ( $V^*$ ):  $S^* = S^*(V^*)$ . This latter function is the real-world – not the ideal – seats-votes curve. The responsiveness ( $r$ ) of this curve tells us how much the seat margin changes for a given incremental change in vote margin. Mathematically, responsiveness ( $r$ ) is the derivative of the function  $S^*(V^*)$  with respect to  $V^*$ :  $r = \frac{dS^*(V^*)}{dV^*}$ ; graphically, it is the slope of the tangent to the seats-votes curve at a specified point. The sensitivity of the efficiency gap to a change in statewide vote margin is captured by the derivative of  $\Delta W$  with respect to  $V$ .

$$\frac{d\Delta W}{dV^*} = r - 2$$

Thus, the sensitivity of the efficiency gap to a change in vote margin depends on the responsiveness of the actual seats-votes curve. If responsiveness is precisely equal to two, the increase in seat share perfectly offsets the increase in vote share and the efficiency gap remains constant. If responsiveness is greater than two, the increase in vote share triggers an overcompensatory increase in seat share, and the efficiency gap increases. If responsiveness is less than two, the increase in vote share triggers an undercompensatory increase in seat share, and the efficiency gap decreases.

#### 7. The Robust, Minimizing Plan

Now consider a mapmaker who knows (or can estimate well) the current statewide vote margin, but who recognizes the vote margin may vary over time with changing electoral circumstances. Suppose this mapmaker wishes to design a plan that is both *minimizing* in the sense that it produces a zero efficiency gap under the current statewide vote margin, and *robust* in the sense that it maintain a zero (or low) efficiency gap if vote margin varies.

How can the mapmaker design such a robust, minimizing plan? The trick is to find a plan that produces a particular seat-vote relationship – one that exhibits the *right ratio* at the current vote margin (double proportionality), and one that flips seats at the *right rate* as vote margin varies (double responsiveness).

Return to our numeric example. Presently, the statewide vote margin is 5%. To produce a zero efficiency gap, the statewide vote margin must be 10%. Thus, Party X must win six out of the 10 seats. And for every 5% change in vote margin, one seat must flip. Assuming uniform swing, this means that the proportional vote margin in each district must be a distinct multiple of 5%. The result is to spread out the competitiveness of each district so that seats flip at the right rate.

DISTRICT	TOTAL VOTES BY PARTY	
	X	Y
1	775	225
2	725	275
3	675	325
4	625	375
5	575	425
6	525	475
7	475	525
8	425	575
9	375	625
10	325	675

Note that only two of ten districts are relatively competitive. District 6 currently favors party x but would flip if party y earned a 5% uniform vote swing; district 7 currently favors party y but would flip if party x earned a 5% uniform vote swing; all other districts would have the same winning party.

C. *False Negatives and False Positives*

The preceding examination reveals how the efficiency gap measure privileges a form of symmetric partisan efficiency over electoral competitiveness and strict seats-votes proportionality. It is not surprising that proponents seek to measure something distinct from competitiveness and proportionality. The Court has repeatedly insisted that the constitution does



not require strict seats-votes proportionality.<sup>126</sup> And in *Gaffney v. Cummings*,<sup>127</sup> the Court was untroubled by the uncompetitiveness of a bipartisan gerrymander that carved the state up into safe Democratic districts and safe Republican districts so as to achieve seats-votes proportionality. Thus, the Court may reject as foreclosed by precedent any legal test that essentially requires proportionality or competitiveness. But this does not mean that proportionality and competitiveness are irrelevant. Strict proportionality may not be required, but a significant departure from proportionality may be relevant to our assessment of a districting plan. So too with districting. A plan need not maximize competitiveness, but we may be rightly concerned if a plan minimizes it. And we ought to be particularly concerned by a plan that departs significantly from both proportionality and competitiveness simultaneously.

If proportionality and competitiveness matter, the efficiency gap's relationship to them present two problems of normative correspondence: the measure may favorably perceive normatively undesirable plans; and the measure may unfavorably perceive normatively desirable plans. Consider each problem in turn.

First, a plan may be gerrymandered in a way the measure cannot detect. A plan may achieve the ideal of equal wasted votes at the expense of both competitiveness and seats-votes proportionality. In this sense, the efficiency gap measure has a significant false-negative problem: it approves plans that exhibit one democratic ideal (equal wasted votes) even if those plans subvert competitiveness, seats-votes proportionality, or both.

Second, the measure may brand as a gerrymander a normatively desirable plan. A plan that produces rough proportionality even when one party enjoys a significant popular majority will necessarily produce a large efficiency gap in favor of the minority party. While both competitive and uncompetitive plans can produce low efficiency gaps, a more competitive plan may present a greater risk of an above-threshold gap in the face of voter swings.<sup>128</sup> In this sense, the efficiency gap measure has a significant false-positive problem: it flags as suspect plans that depart from the ideal of equal

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<sup>126</sup> See, e.g., *Vieth v. Jubelirer*, 541 U.S. 267, 288 (2004) (plurality) (“[Appellants’] standard rests upon the principle that groups...have a right to proportional representation. But the Constitution contains no such principle. It guarantees equal protection of the law to persons, not equal representation in government to equivalently sized groups.”)

<sup>127</sup> 412 U.S. 735 (1973).

<sup>128</sup> A very uncompetitive plan will produce an above threshold gap only with a 4% swing. But that above-threshold gap will not be durable under sensitivity analysis. A very competitive plan can produce a zero gap, but a small swing can produce an above threshold gap. But that above-threshold gap will not be durable under sensitivity analysis. There is uncertainty about whether and how sensitivity analysis will play a role. Both a very competitive and a very uncompetitive plan can exhibit a zero gap at expected vote share but an above-threshold gap with vote swings. But the necessary vote swing will be smaller for the competitive plan, and so the competitive plan will have a higher risk of producing an above threshold gap. Will mapmakers prefer to avoid generating unstable above threshold gaps? If so, they may prefer less competitive plans.

wasted votes, even if that departure reflects an effort to promote democratic norms like competitiveness and seats-votes proportionality.

By privileging the ideal of equal wasted votes, the efficiency gap measure not only favors some plans that subvert competitiveness and proportionality (the false-negative problem), but also disfavors some plans that promote competitiveness or proportionality (the false-positive problem). For these reasons, the efficiency gap measure may promote fairness in the sense of symmetric partisan efficiency in a way that unintentionally encourages uncompetitive elections that accords majorities disproportional to their popular support.

The efficiency gap measure consists not only of this measure, but also sensitivity analysis, intent, and justification elements. These other doctrinal tools mitigate but do not eliminate the false positive problem. If a highly competitive plan produces a large efficiency gap in one election, that gap will likely be unstable under sensitivity analysis. And a plan that promote competitiveness or proportionality is more likely to be justified by legitimate districting principles and less likely to be branded the result of discriminatory intent. But these tools do not eliminate the problem. Mapmakers may fear not only liability, but also litigation. An above-threshold gap may be unstable, unintended or justified, but it is likely to invite legal challenge, and may only prevail after the considerable delay, expense, and risk of a trial before a three-judge federal panel. And these tools do not address the false negative problem, because proponents offer no mechanism to overcome the presumption of validity triggered by a below threshold gap. If the Court embraced the efficiency gap as the definitional measure of partisan gerrymandering, plans that consistently produced low efficiency gaps would be judicially bullet-proof. This would be problematic in jurisdictions where one party enjoyed a large majority. This party could achieve a low gap by carving the state up into safe districts that accorded it most or all of the seats. One way to address this concern would be to develop a separate legal test for bipartisan gerrymanders.

#### **IV. GENERALIZING THE EFFICIENCY GAP**

Part II clarified that the efficiency gap measure relies on electoral assumptions and methodological choices. The long-form calculation adopts a particular definition and weight of surplus votes and an absolute scale of comparison and ignores uncontested races and peripheral support. The simplified formula relies on the further assumption of equal voter turnout. This Part systematically relaxes some of these electoral assumptions and methodological choices, deriving more generalized formulas that quantify the precise impact of these assumptions and choices. Part A demonstrates that the definition and weight of surplus votes determines the measure's relationship with competitiveness and seats-votes proportionality. Part B considers a more voter-centric approach that idealizes equal wasted vote

shares rather than equal wasted vote totals. Part C briefly discusses the problem of uncontested seats. Part D examines the impact of voter turnout and demonstrates that voter suppression may perversely reduce the efficiency gap.

A. *Definition and Weight of Surplus Votes*

Both the *definition* and the *weight* of surplus votes are methodological choices – not self-defining concepts. They are susceptible to competing interpretations, and selecting among them requires deliberation and transparency. The proponents and litigants define a surplus vote using a threshold of half of actual voter turnout; and they equally weight lost and surplus votes.<sup>129</sup> But neither the proponents nor the litigants have adequately explained these two distinct but related methodological choices.

The proponents simply define a wasted vote as a lost *or* surplus vote,<sup>130</sup> assuming implicitly and without explanation that lost and surplus votes should be equally weighted. But when considered from the perspective of an individual voter, lost and surplus votes are not obviously equivalent. True enough, both the voter who casts the lost vote and the voter who casts the surplus vote may regret that their vote could have been more effective in another district. But the voter who casts a surplus vote gets to be represented by the candidate of her choice. Not so with the voter who casts a lost vote. Faced with a choice between casting a lost vote and casting a surplus vote, I would prefer the latter option, and I suspect most other voters would, too.

The choice to define surplus votes as half the vote margin is similarly unexplained and even less intuitive. The proponents define a surplus vote as one “cast for a winning candidate but *in excess of what she needed to prevail*”<sup>131</sup> but then describe it as “any vote beyond the 50 percent threshold needed (in a two-candidate race) to win a seat.”<sup>132</sup> But these alternative linguistic formulations are not identical – a point the dissenting judge in the *Whitford* case makes with a baseball analogy:

Just as a baseball game is not decided by reference to total runs, an election is not decided by a fraction of total votes. Instead, the number of votes needed to win is simply the number one more than the losing candidate won, and therefore anything beyond that should be counted as a “wasted” vote.<sup>133</sup>

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<sup>129</sup> See *supra* Part II.A.

<sup>130</sup> Stephanopoulos & McGhee, *supra* note 7, at 851.

<sup>131</sup> *Id.* at 834, 851 (emphasis supplied).

<sup>132</sup> *Id.* at 851.

<sup>133</sup> *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*93 (W.D. Wis. Nov. 21, 2016) (Griesbach, J., dissenting).

On this basis, the dissent attacked the definition of surplus votes as “opaque”<sup>134</sup> and “absurd,”<sup>135</sup> suggesting that surplus votes must be defined as the entire vote margin rather than half the vote margin.<sup>136</sup> Recently, Professor John Nagle has also noted that surplus votes can be alternatively defined as the entire vote margin,<sup>137</sup> and that wasted votes can be generalized as a weighted sum of lost and surplus votes.<sup>138</sup>

The proponents definition of surplus votes is not absurd, though it could be explained more clearly. It simply defines surplus votes as the number of voter swaps possible without altering the outcome. Because each swap exchanges one x-supporter for one y-supporter, it decreases the victory margin by two votes, so the total number of possible outcome-preserving swaps is half the victory margin. This reflects the perspective of the mapmaker under the equal voter turnout assumption. Each voter swap represents a marginal adjustment to the district boundaries – one that changes the district of only the two homes where the respective swapped voters reside. This definition of surplus votes is not absurd, but it does privilege the party-centric perspective of the mapmaker under the stylized assumption of equal voter turnout.

This Section provides a fuller explanation of both the rationales for and consequences of these methodological choices. Using parameters I call *alpha*, *beta*, and *gamma*, I relax both the definition and weight of surplus votes to show how adopting competing methodological approaches would affect the democratic norms of competitiveness and proportionality. I use the term “parameter” to denote a variable that an analyst *selects*, rather than a variable that an analyst *observes*. Each parameter – *alpha*, *beta*, and *gamma* – represents a real number that an analyst may choose.

*Alpha* ( $\alpha$ ) captures *weighting* of surplus votes. The efficiency gap as proposed measures the number of wasted votes through the equation:

$$W_{pi} = L_{pi} + E_{pi}$$

In order to weight lost and excess votes differently, we can simply multiply the number of excess votes by the parameter *alpha* ( $\alpha$ ), which we can set variably to capture how heavily we wish to weight excess votes as compared to lost votes. When we add *alpha* ( $\alpha$ ), the wasted-votes equation becomes:

$$W_{pi} = L_{pi} + \alpha E_{pi}$$

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<sup>134</sup> *Id.* at \*92 (Greisbach, J., dissenting).

<sup>135</sup> *Id.* at \*93 (Greisbach, J., dissenting).

<sup>136</sup> *Id.* (Greisbach, J., dissenting).

<sup>137</sup> John F. Nagle, *How Competitive Should a Fair Single Member Districting Plan Be?*, 16 ELECTION L.J. \_\_\_, 4 & n.16, 8 & n.24 (2017).

<sup>138</sup> *Id.* at 5.

Under the proponents' definition, *alpha* ( $\alpha$ ) is set to one. If, alternatively, we set *alpha* ( $\alpha$ ) to zero, then we would ignore surplus votes entirely. If we set *alpha* ( $\alpha$ ) to 0.5, then we would weight surplus votes half as heavily as lost votes. If we set *alpha* ( $\alpha$ ) greater than one, we weight surplus votes more heavily than lost votes.

*Beta* ( $\beta$ ) captures the *definition* of surplus votes. The efficiency gap as proposed equates the number of surplus votes as one-half of the margin of victory:

$$E_{pi} = \frac{1}{2} M_i$$

But surplus votes could also be plausibly defined as the *entire* margin of victory. Thus we can replace the number 0.5 with the parameter *beta* ( $\beta$ ) to allow an analyst to vary the definition of a surplus vote. If we do so, we get the following generalized equation defining surplus votes:

$$E_{pi} = \beta M_i$$

We can set *beta* ( $\beta$ ) to one half (using proponents' definition) or to one (using the alternative definition) – or at some intermediate value.

*Gamma* ( $\gamma$ ) captures the combined impact of weight and definition. *Gamma* ( $\gamma$ ) is simply the product of *alpha* and *beta*.

$$\gamma = \alpha\beta$$

We can now repeat all the relevant steps involved in the unpacking and reframing of the efficiency gap measure, but this time in terms of these parameters to quantify the precise impact of the way we define and weight surplus votes. The definition-generalized efficiency gap is:

$$\Delta W_{xy}(\gamma) = \frac{W_y - W_x}{V_y + V_x}$$

$$\text{where } W_p = \sum_D W_{pi}$$

$$\text{and } W_{pi} = \begin{cases} \gamma M_i & i \text{ in } D_p \\ V_{pi} & i \text{ in } D'_p \end{cases}$$

The definition-generalized simplified formula is:

$$S^* - r^*(\gamma)V^*$$

where  $r^* = 1 + 2\gamma$

Define the generalized minimizing proportional margin of victory  $m^*(\gamma)$  as the following function of parameter  $\gamma$ :

$$m^*(\gamma) = \frac{1}{r^*(\gamma)}$$

Define the generalized competitiveness score in terms of this new minimizing margin of victory

$$c_i(\gamma) = m^*(\gamma) - m_i$$

And the definition-generalized competitiveness gap is:

$$\Delta C_{xy}(\gamma) = S_x C_x(\gamma) - S_y C_y(\gamma)$$

$$\text{where } C_p(\gamma) = \frac{\sum d_p c_i(\gamma)}{d_p}$$

$$c_i(\gamma) = m^*(\gamma) - m_i$$

$$m^*(\gamma) = \frac{1}{r^*(\gamma)}$$

With these new definitions, under equal voter turnout, the efficiency gap, competitiveness gap, and shorthand formula are all still equal.<sup>139</sup>

$$\Delta W_{xy}(\gamma) = S^* - r^*(\gamma)V^* = \Delta C_{xy}(\gamma)$$

The measure still captures relative wasted votes, undeserved vote share, and differential competitiveness. The measure still generally privileges equal wasted votes over competitiveness and seats-votes proportionality. But the precise trade-offs between these competing norms is determined by the precise values the analyst selects for parameters *alpha* and *beta*. The following table summarizes the effects of varying these two parameters.

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<sup>139</sup> See Technical Appendix, Part X.

Table 3.

	Weight of surplus vote relative to lost vote	Relationship between surplus votes and vote margin	Product of alpha and beta	Golden plan ratio between statewide vote margin and statewide seat margin	Generalized Simplified Formula For Plan-Level Efficiency Gap	Golden district vote margin	Golden district vote share	Statewide vote share that enables total capture with zero gap	Statewide vote share that enables (50 + S) supermajority with sub-threshold gap
Parameter	$\alpha$	$\beta$	$\gamma$	$r^*$	$G$	$m^*$	$v^*$	$V_{100.0}$	$[V_{100.0}, V_{87.8}, V_{100.0}]$
Equation				$1 + 2\gamma$	$S - r^*V$	$\frac{1}{r^*}$	$\frac{1 + m^*}{2}$	$100v^*$	$\frac{100v^* - (100 - S + G)m^*}{2}$
<b>OPTION 1</b> (Alternative Definition)	1	1	1	3	$S - 3V$	1/3	2/3	67	64,60,56
<b>OPTION 2</b> Traditional Interpretation	1	1/2	1/2	2	$S - 2V$	1/2	3/4	75	71,65,59
<b>OPTION 3</b> Alternative Interpretation	1/2	1	1/2	2	$S - 2V$	1/2	3/4	75	71,65,59
<b>OPTION 4</b> Alternative Definition & Weighting	1/2	1/2	1/4	3/2	$S - \frac{3}{2}V$	2/3	5/6	87	81,73,65
<b>OPTION 5</b> Alternative Weighting	0	1	0	1	$S - V$	1	1/1	100	92,79,67

As this table shows, the parameter *gamma* ( $\gamma$ ) is like a dial the analyst can turn to calibrate the relationship between the efficiency gap, competitiveness, and seats-votes proportionality. When *gamma* is one, the minimizing district is won 67-33, mitigating the competitiveness problem, but the golden plan exhibits triple seats-votes proportionality, exacerbating the proportionality problem. When *gamma* is zero, the minimizing district is won 100-0, exacerbating the competitiveness problem, but the golden plan exhibits strict seats-votes proportionality, eliminating the proportionality problem. The proponents eschew either of these “pure” approaches. Rather than turning the dial all the way in one direction or the other, they adjust the dial to an intermediate position, setting *gamma* equal to one half. This intermediate calibration avoids the most extreme tension with either norm, opting instead for more moderate tension with both norms.

There are two mathematically equivalent ways to set *gamma* equal to one half: (1) defining a surplus vote as half the vote margin while equally weighting lost and surplus votes; or (2) defining a surplus vote as the entire vote margin while weighting a surplus vote half as heavily as a lost vote. In the latter approach, the definition is more intuitive, but the weight is more arbitrary and therefore harder to discern and justify. Proponents opt instead for the former approach, adopting a more intuitive weight but a less intuitive definition. In this sense, defining and weighting surplus votes involves a trade-off not only between competitiveness and seats-votes proportionality, but between *conceptual* correspondence and *evaluative* correspondence.

### B. Totals Versus Shares

In criticizing the proponent’s approach on discernibility grounds, McGann suggested that each party should waste the same *share* of votes,

rather than the same *number* of votes.<sup>140</sup> Suppose 6,000,000 votes are cast, 4,000,000 for party x and 2,000,000 for party y and suppose each party wastes 1,500,000. This means that 75% of ballots cast by party y supporters are wasted, while only 37.5% of ballots cast by party x supporters are wasted. Each party wastes the same number of aggregate votes, but a party y supporter is twice as likely as a party x supporter to waste her vote.

Following this suggestion, Nagle has developed what he calls a “voter-centric” measure that compares relative wasted vote shares rather than relative wastes votes as a proportion of all ballots cast. However, Nagle defined a surplus vote as half the victory margin rather the full victory margin, noting that the alternative was mathematically equivalent to unequal weighting of lost and surplus votes. Nagle then rejected unequal weighting of lost and surplus votes on the ground that it would violate McGhee’s efficiency principle.<sup>141</sup> In this section I suggest the plausibility and superiority of a measure that compares shares rather than absolute numbers of wasted votes, with a surplus vote defined as the full vote margin and lost and surplus votes weighted equally. The measure is more discernible within the prevailing individual rights framework because it is voter rather than party centric in both its scale and its definition of surplus vote. And the measure is more structure-resonant because it coheres more closely with electoral reality and with the democratic values of seats-votes proportionality and competitiveness.

Consider a modified efficiency gap measure with a voter-centric scale and definition of surplus vote.

$$\Delta W^V_{xy} = \frac{W_y}{V_y} - \frac{W_x}{V_x}$$

$$\text{where } W_p = \sum_D W_{pi}$$

$$\text{and } W_{pi} = \begin{cases} \gamma(V_{pi} - V_{2i}) & D_p \\ V_{pi} & D'_p \end{cases}$$

Recall that we can express vote totals, and thus wasted votes, in terms of victory margin and turnout.

$$W_{pi} = \begin{cases} \gamma T_i m_i & D_p \\ \frac{T_i}{2} (1 - m_i) & D'_p \end{cases}$$

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<sup>140</sup> McGann p. 296

<sup>141</sup> Nagle 2016 at 8 (“However, as Eric McGhee has kindly pointed out, the possibility that different values of S for the same vote V may give the same value of bias violates a fundamental principle for bias measures (McGhee 2016), namely, gerrymandering might be able to increase S for the same V and not be detected by the measure of bias.”)



$$W_p = \sum_D W_{pi} = \sum_{D'_p} \frac{T_i}{2} (1 - m_i) + \sum_{D_p} \gamma T_i m_i$$

By applying equal voter turnout ( $T_i = T$ ),  $V_x = dTV$ ,  $V_y = dT(1 - V)$ ,  $\frac{d_x}{d} = S$ , and  $\frac{d_y}{d} = S$ , and we can express each party's wasted vote share as a function of party-x seat share ( $S$ ) and party-x vote share ( $V$ ), the parameter  $\gamma$ , and the average victory margin in x-won and y-won districts,  $m_x$  and  $m_y$ .

$$\frac{W_y}{V_y} = \frac{\sum_{D_x} \frac{T}{2} (1 - m_i) + \sum_{D_y} \gamma T m_i}{dT(1 - V)} = \frac{2S - 2Sm_x + 2\gamma 2(1 - S)m_y}{4(1 - V)} \quad (\text{EQ1})$$

$$\frac{W_x}{V_x} = \frac{\sum_{D_x} \gamma T m_i + \sum_{D_y} \frac{T}{2} (1 - m_i)}{dT V} = \frac{2\gamma 2Sm_x + 2(1 - S) - 2(1 - S)m_y}{4V} \quad (\text{EQ2})$$

Nagle proceeded as though  $m_x$  and  $m_y$  were independent variables.<sup>142</sup> But recall that the seat-share weighted sum of  $m_x$  and  $m_y$  is the average victory margin over all districts  $m$  and the seat-share weighted difference of  $m_x$  and  $m_y$  is twice the vote margin.

$$Sm_x + (1 - S)m_y = m \quad (\text{EQ3})$$

$$Sm_x - (1 - S)m_y = 2V - 1 \quad (\text{EQ4})$$

Rearranging, we can express  $m_x$  and  $m_y$  in terms of  $m$ ,  $V$  and  $S$ .

$$2Sm_x = m + 2V - 1 \quad (\text{EQ5})$$

$$2(1 - S)m_y = m - 2V + 1 \quad (\text{EQ6})$$

Substituting (EQ5) and (EQ6) into (EQ1) and (EQ2) and simplifying, we can express each party's wasted vote share as a function of  $S$ ,  $V$ ,  $\gamma$ , and  $m$ .

$$\frac{W_y}{V_y} = \frac{2S - m(1 - 2\gamma) - 2V(1 + 2\gamma) + 1(1 + 2\gamma)}{4(1 - V)} \quad (\text{EQ7})$$

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<sup>142</sup> Nagle called his parties A and B instead of X and Y and focused on the “the average A vote for those districts won by A and the average A vote for those districts won by B.” 2016:13 (Appendix B). These two values are related to the parties' respective average victory margins. Only after setting equal the parties' respective wasted vote shares did Nagle conclude that these two values are “are not independent; [one] can be determined from [the other] through a quadratic formula.” 2016:13 (Appendix B). However, even when respective wasted vote shares are not equal, the two values are related to one another through equations () and ().

$$\frac{W_x}{V_x} = \frac{-2S - m(1-2\gamma) + 2V(1+2\gamma) + 1(1-2\gamma)}{4V} \quad (\text{EQ8})$$

The new measure ( $\Delta W^V_{xy}$ ) is the difference between each party's wasted vote share. Subtracting (EQ8) from (EQ7) and simplifying:

$$\Delta W^V_{xy} = \frac{2S - 2V(1+2\gamma) - (1-m(1-2\gamma))(1-2V) + 2\gamma}{4V(1-V)} \quad (\text{EQ9})$$

Setting  $\Delta W^V_{xy}$  to zero yields the following ideal seats-votes formula:

$$S = (1 + 2\gamma)V + \frac{(1 - m(1 - 2\gamma))}{2}(1 - 2V) - \gamma$$

Expressed in terms of seat margin  $\bar{S} = S - \frac{1}{2}$  and vote margin  $\bar{V} = V - \frac{1}{2}$ , this ideal seats-votes formula is:

$$\bar{S} = (2\gamma - m(2\gamma - 1))\bar{V}$$

With equal weighting of lost and surplus votes ( $\alpha = 1$ ) and the proponents' definition of surplus votes based on half the vote margin ( $\beta = \frac{1}{2}$ ), parameter  $= \frac{1}{2}$ , then the competitiveness term  $m$  drops out and the ideal seats-votes relationship reduces to strict proportionality.

$$\bar{S}\left(\gamma = \frac{1}{2}\right) = \left(2\frac{1}{2} - m\left(2\frac{1}{2} - 1\right)\right)\bar{V} = (1 - m(0))\bar{V} = \bar{V}$$

This result accords with Nagle.<sup>143</sup> However, if we maintain equal weighting of lost and surplus votes ( $\alpha = 1$ ) and but alternatively define surplus votes based on the entire vote margin ( $\beta = 1$ ), parameter  $= 1$ , then the competitiveness term  $m$  remains and the ideal seats-votes relationship reduces to the following:

$$\bar{S}(\gamma = 1) = (2 * 1 - m(2 * 1 - 1))\bar{V} = (2 - m)\bar{V}$$

Under maximal uncompetitiveness, when each prevailing candidate earns all the votes ( $m \rightarrow 0$ ), this reduces to strict proportionality. Under maximal competitiveness, when each prevailing candidate wins by a single vote ( $m \rightarrow 1$ ), this reduces to the same double proportionality that emerges from

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<sup>143</sup> 2016: 6 eq. (6)

the proponent's party-centric approach based on equal wasted vote totals (not shares) and surplus votes defined based on half (not all) of the victory margin. But unlike the proponent's approach, this seats-votes curve depends on the competitiveness term  $m$ . As  $m$  varies from 0 to 1, the slope varies from 2 to 1. The seat bonus is still capped at two, but now the majority party can only achieve that maximal seat bonus if it maximizes competitiveness. If the system is less than maximally competitive, the seat bonus must be less than two.

Nagle recognized that his voter-centric approach under an alternative definition of surplus votes<sup>144</sup> would produce an ideal seats-votes curve that depended on competitiveness.<sup>145</sup> But Nagle, following McGhee, views this dependence on competitiveness as an undesirable – indeed fatal – feature of the measure under the alternative definition of surplus votes ( $\gamma \neq \frac{1}{2}$ ).

However, as Eric McGhee has kindly pointed out, the possibility that different values of  $S$  for the same vote  $V$  may give the same value of bias violates a fundamental principle for bias measures (McGhee 2016), namely, gerrymandering might be able to increase  $S$  for the same  $V$  and not be detected by the measure of bias. The mechanism to do this is to draw the lines to change the average district votes. Making districts more competitive allows a gerrymandering party that has  $V > \frac{1}{2}$  to increase its  $S$  with no change in this measure of bias when  $[\gamma > \frac{1}{2}]$ .

Nagle and McGhee are right that when  $\gamma = 1$  or more generally  $\gamma > \frac{1}{2}$ , the majority party can increase its seat share while maintaining constant vote share and equal wasted vote shares by increasing average districting competitiveness. And this property violates McGhee's strict efficiency principle, which defines efficiency as increasing seat share at constant vote share. But this property does not violate a modified efficiency principle, which defines efficiency as increasing expected seat share at constant vote share. This is because the majority party can only increase its seat share at current vote share by increasing the competitiveness of the system, which makes the outcome less robust to vote swings.

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<sup>144</sup> Nagle used the proponents' definition of surplus votes as half the victory margin, but noted that the alternative definition was equivalent to doubling the relative weight of surplus votes. See 2016: 4 n. 16 and 8 n. 24.

<sup>145</sup> "Making districts more competitive allows a [majority] party...to increase its [seat share] with no change in this measure..." [2016: 8]

But there is another way to address the measure's dependence on competitiveness when  $\gamma > \frac{1}{2}$ . Rather than using the system's actual competitiveness ( $\mathbf{m}$ ), the analyst can compare the difference in wasted vote shares that would have obtained if the system exhibited an ideal baseline level of competitiveness ( $\hat{\mathbf{m}}$ ). Political scientists estimate that real-world seats-votes curves tend to exhibit competitiveness that varies with vote share, exhibiting high competitiveness when the majority enjoys a modest vote margin and low competitiveness when the majority enjoys a significant vote margin. This produces an s-shaped curve that is relatively flat far away from the (0.5,0.5) point where each party earns half the votes but relatively steep near the (0.5,0.5) point. This accommodation between competitiveness and proportionality favors the majority when the minority is large but protects the minority when it is small. Because the seat bonus decreases with majority vote share, the majority must earn almost all the votes in order to win all the seats.

This suggests an alternative wasted vote measure. First, construct a competitiveness measure  $\hat{\mathbf{m}}(\bar{V})$  that aligns with these sensible features of electoral reality. Second, compute the difference in the parties' wasted vote shares (not totals), using the *full* (not half) definition of surplus votes ( $\gamma = 1$ ), that would obtain under  $\hat{\mathbf{m}}(\bar{V})$ . An appropriate competitiveness measure would satisfy the following properties:

$$\hat{\mathbf{m}}(\mathbf{0}) = \mathbf{0} \quad \text{Maximal competitiveness at minimal vote margin}$$

$$\hat{\mathbf{m}}\left(\frac{1}{2}\right) = \mathbf{1} \quad \text{Minimal competitiveness at maximal vote margin}$$

$$\hat{\mathbf{m}}(\bar{V}) = \hat{\mathbf{m}}(-\bar{V}) \quad \text{Symmetric treatment of parties}$$

The simplest function that satisfies these properties is:<sup>146</sup>

$$\hat{\mathbf{m}}(\bar{V}) = 4\bar{V}^2$$

If a plan exhibited this level of competitiveness, each party would waste equal vote share when the following relationship obtained:

$$\bar{S}(\gamma = 1) = (2 - \hat{\mathbf{m}}(\bar{V}))\bar{V} = (2 - (4\bar{V}^2))\bar{V} = 2\bar{V} - 4\bar{V}^3$$

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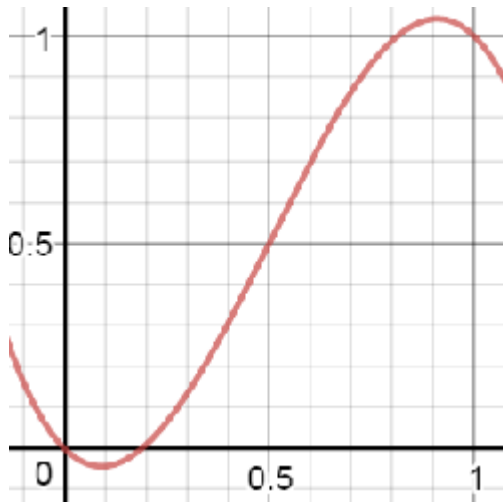
<sup>146</sup>  $\hat{\mathbf{m}}(\mathbf{0}) = 4 * \mathbf{0} = \mathbf{0}$

$\hat{\mathbf{m}}\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 = 4\left(\frac{1}{4}\right) = \mathbf{1}$

$\hat{\mathbf{m}}(-\bar{V}) = 4(-\bar{V})^2 = 4\bar{V}^2 = \hat{\mathbf{m}}(\bar{V})$

$$\bar{S} = 2\bar{V} - 4\bar{V}^3$$

Here is a graph of this curve<sup>147</sup>:



This measure may be more discernible, as it adopts more intuitive, voter-centric approaches to scale and definition. And it may avoid the extreme vote share problem, where a majority can completely (or effectively) shut out a minority and capture all (or most) of the seats while maintaining a zero gap. And this measure satisfies McGhee’s strict efficiency principle. The drawback, of course, is that it relies on a baseline competitiveness measure the analyst selects, which introduces conjecture. However, the analyst could derive this relationship from real election data. Thus the baseline would be electoral reality of what the relationship generally is, not the analyst’s subjective judgment about what the relationship ought to be. A large gap would indicate that the majority has earned significantly more seats than the majority would have if the plan equalized the parties’ respective wasted vote shares under a level of competitiveness consistent with the generally prevailing electoral relationship between competitiveness and vote share.

### C. *Uncontested Races and Ballot-Dependence*

The first electoral circumstance assumed by the efficiency gap proposal is that every race is contested – every race is a competition between one candidate from Party X and one candidate from Party Y. The proponents are explicit about this assumption, and they candidly recognize that this assumption does not match up with real-world election data. To deal with this

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<sup>147</sup> Source: Desmos Graphic Calculator

disconnect, the proponents advance the strategy of *imputation* of election results in uncontested races:

Going forward, we encourage other scholars to explore a range of imputation techniques to ensure that the direction of a gerrymander (if not its size) is robust to any particular strategy. But this catholic philosophy has its limits. We strongly discourage analysts from either dropping uncontested races from the computation or treating them as if they produced unanimous support for a party. The former approach eliminates important information about a plan, while the latter assumes that coerced votes accurately reflect political support. Neither correctly represents how the gerrymandering party itself would view its plan.<sup>148</sup>

Uncontested elections present problems of correspondence, robustness, and scope for the efficiency gap. If no imputation is permitted, we could either omit the uncontested election from the data set (presenting a problem of scope) or count the uncontested district and register it as indicating unanimous support for the winning party (presenting a problem of normative correspondence). If imputation is permitted, we must choose a fair and accurate method of imputation (presenting a challenge of robustness).

The need to impute election results in uncontested districts prompts three analytical points related to our overall assessment of the efficiency gap measure. First, the imputation method chosen may, under the right circumstances, have a significant, and even outcome-determinative, impact on the measure of a plan's efficiency gap. Second, if the efficiency gap measure permits and even incentivizes uncompetitive electoral maps, then the percentage of districts that are uncontested – and hence the problems posed by imputing election results – may increase. Third, the recognition that the efficiency gap measure does not register gerrymandering dynamics in the context of uncontested elections has broader implications about the measure's correspondence to the multiple normative concerns implicated by gerrymandering.

My first point is practical. Proponents recognize that employing different imputation methods may present robustness problems but do not quantify how significantly the choice of imputation method might impact the resulting efficiency gap. It is possible, however, to mathematically estimate how sensitive the gap is to different imputation approaches using the simplified formula. When an analyst imputes vote share for an uncontested election, she changes only the vote totals, not the winning party. The seat margin stays the same, but the vote margin changes. Imagine two analysts employing different imputation methods that produce different imputed vote

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<sup>148</sup> Stephanopoulos & McGhee, *supra* note 7, at 867.

margins in uncontested races. Let  $S_U$  denote the share of districts with uncontested seats,  $V_{1U}$  the average vote share for party x in uncontested races as imputed by the first analyst,  $V_{2U}$  the average vote share for party x in uncontested races as imputed by the second analyst, and  $\Delta W_{12}$  the difference between the efficiency gap computed by the first analyst and the efficiency gap computed by the second analyst. Then we can quantify the impact of imputation under the equal voter turnout assumption as follows:<sup>149</sup>

$$\Delta W_{12} = -2S_U(V_{1U} - V_{2U})$$

The greater the difference between imputed average vote shares, the greater the difference in computed gaps. Note that the analyst who imputes the higher average vote share will compute the lower gap. By increasing the estimate of party x support, the analyst decreases the estimate of the extent to which the plan favors party x. The greater the share of districts with uncontested races, the greater the difference in computed gaps. The more uncontested races, the more imputation technique matters. For example, assume that one-third of all districts in a state held uncontested elections. Next, assume that two different analysts imputed election results. Assume that Analyst A concluded that Party X would have, on average, earned 70% of the vote in uncontested races, but Analyst B concluded that Party X would have, on average, won 73% of the vote in uncontested races. Analyst A would compute an efficiency gap 2% higher than analyst B.

This effect of the imputation method upon the statewide efficiency gap is no mere theoretical concern. The choice of imputation method could have significant practical consequences for the overall validity of the plan. In the Wisconsin case, for example, the plan's efficiency gap only exceeded the numeric threshold by 2 to 5%.<sup>150</sup> If enough of the districts' elections were uncontested, it is entirely plausible that a two-percent drop in the efficiency gap could be achieved by employing an alternative imputation method. And in Wisconsin in 2012, twenty-three Democratic-won races (about 23% of the races overall) were uncontested and therefore imputed.<sup>151</sup> There might be

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<sup>149</sup>  $\Delta W_{12} = [S - 2(V_C + V_{1U})] - [S - 2(V_C + V_{2U})] = -2S_U(V_{1U} - V_{2U})$

<sup>150</sup> Experts in the case disagreed on the appropriate calculation of the gap and the appropriate numeric threshold. See *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*51 (W.D. Wis. Nov. 21, 2016).

<sup>151</sup> "Twenty three Democratic candidates were uncontested, indicating a significant level of packing (the bar at the far left side of the figure); uncontested races occur largely when one party sees zero probability of winning because the majority party has such overwhelming majorities in the district. By contrast, only four Republicans were uncontested." Expert Report of Kenneth R. Mayer, 3:15-cv-00421-bbc No. 54, at 40 (Jan. 5, 2016), available at <http://www.campaignlegalcenter.org/sites/default/files/Mayer-WHITFORD%20V.%20NICHOL-Report.pdf>. See also Expert Report of Simon Jackman, 3:15-cv-00421-bbc No. 62, at 22 (Jan. 25 2016) available at [http://www.campaignlegalcenter.org/sites/default/files/Jackman-WHITFORD%20V.%20NICHOL-Report\\_0.pdf](http://www.campaignlegalcenter.org/sites/default/files/Jackman-WHITFORD%20V.%20NICHOL-Report_0.pdf) ("For 38.7% of the district-level results in this analysis, it isn't possible to directly compute a two-party vote share (vi), either because

many real-world cases in which the existence *vel non* of an above-threshold gap depends on the imputation method used for uncontested races.<sup>152</sup> If the efficiency gap proposal were adopted as a legal test, plaintiffs alleging a political gerrymander may strategically employ imputation methods that would produce larger gaps; defendants may strategically employ competing imputation methods; and the battle of the experts could be outcome-determinative.

With the imputation of hypothetical election results, one of the primary attractive features of the efficiency gap measure becomes muddled. The efficiency gap may be particularly appealing – especially to Justice Kennedy – because it relies upon directly observed election data, not hypothetical results.<sup>153</sup> But if calculating the gap requires imputing hypothetical election results, and if the size of the gap depends on which imputation method the analyst selects, the gap is less a straightforward measure of real-world data.

My second point is predictive. If, as I have explained earlier, adopting the efficiency gap as the exclusive measure of political gerrymandering would fail to constrain – and may in fact incentivize – uncompetitive bipartisan gerrymanders, then the problems associated with uncontested races will only increase. An uncontested race is a signal that a district is highly uncompetitive.<sup>154</sup> Adoption of the efficiency gap measure as the definition of gerrymandering could promote uncompetitive races, leading to uncontested races, making the measure less directly tied to observed data and more susceptible to a battle of the experts (and their respective imputation methods).

My third point is normative. By endorsing a strategy of imputation to deal with uncontested races, proponents acknowledge that in one limited situation – that of uncontested races – the raw inputs relied on in calculating the gap are incomplete. In this limited case, there is a recognition that the number of ballots cast does not accurately reflect popular support amongst the people of that district for the two different parties; the election results are not a referendum on party control because there was no choice between parties. This recognition, however, is limited to this one extreme situation of uncontested races. Analogously, as described above, proponents try to limit the statewide “extreme vote share problem” to a single point of districts won 75 to 25, rather than recognizing it is a feature of the system that extends beyond that one point.

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the seat was uncontested or not contested by both a Democratic and Republican candidate, or (in a tiny handful of cases) the data are missing.”)

<sup>152</sup> “Uncontested races are common in state legislative elections, and are even the norm in some states. Expert Report of Simon Jackman, *supra* note 140, at 22.

<sup>153</sup> *LULAC v. Perry*, 548 U.S. 399, 419–20 (2006) (opinion of Kennedy, J.).

<sup>154</sup> “[U]ncontested races occur largely when one party sees zero probability of winning because the majority party has such overwhelming majorities in the district.” Expert Report of Kenneth R. Mayer, *supra* note 140, at 40.



Here, too, the problem perceived in the context of uncontested elections – that the number of ballots actually cast imperfectly captures underlying political support – extends more generally beyond this scenario. Most obviously, it extends to other scenarios in which an election is formally – but only lightly – contested. But more broadly, district elections are not perfect referenda on party control, and the number of ballots cast does not capture all of the information that would be relevant to the existence of a political gerrymander – not just in uncontested races but in a whole host of situations for a whole host of reasons. The dissenting judge in *Whitford*,<sup>155</sup> as well as political scientists,<sup>156</sup> have recognized that more generally a district race is partly about party preference, but partly about which individual representative voters prefer. Sometimes, in fact, candidates switch parties and still retain their seats – because their voters are choosing to retain *that incumbent*, not sticking with that party.<sup>157</sup>

If we focus only on ballots cast – as the efficiency gap measure does – problems of scope and correspondence arise. The efficiency gap measure cannot capture, for instance, nonpartisan races at all (as often occur at the local level). More generally, because the efficiency gap measures only ballots cast, without considering other indicators of problematic political manipulation of elections, it does not register (and may actually incentivize) other normatively harmful political manipulation – including voter suppression efforts. The next section explores this dynamic in more depth.

#### *D. Turnout*

The long-form definition of the efficiency gap as relative wasted votes does not rest on the assumption of equal voter turnout across districts, but the proponents explicitly introduce that assumption in deriving the simplified formula. If we relax the assumption of equal voter turnout, two primary observations can be drawn.

First, when scrutinized, the assumption that voter turnout is, in fact, close to equal across districts, is actually quite implausible. This raises concerns about the degree of divergence of results calculated using the long-form formula and those calculated using the simplified formula – particularly when analyzing voluminous historical election data to identify a numeric threshold. To assist in understanding the relationship between the long-form calculation and the short-form calculation, I explain that in fact the assumption of equal voter turnout is a *sufficient* but not a *necessary* assumption to derive the simplified formula. Instead, if *average differential*

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<sup>155</sup> *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*91 (W.D. Wis. Nov. 21, 2016) (Griesbach, J., dissenting).

<sup>156</sup> Krasno, *supra* note 16, at 5.

<sup>157</sup> See, e.g., *Teton Cty. Republican Cent. Comm. v. Hansen*, No. 4:16-CV-00160-CWD, 2016 WL 1574028, at \*1 (D. Idaho Apr. 19, 2016) (permitting Idaho Sheriff elected as a Democrat to run for reelection as a Republican).

voter turnout is equal between X-won districts and Y-won districts, the results obtained by using the simplified and long-form calculations will be the same. Otherwise, the turnout gap will determine the difference between the simplified and long-form calculations.

Second, upon closer analysis, we can see that levels of voter turnout impact the calculation of the long-form efficiency gap. When we understand the dynamics of this relationship, a concern emerges that adoption of the efficiency gap measure as a legal standard may unintentionally incentivize and reward voter suppression.

### 1. Relaxing the Equal Voter Turnout Assumption

McGhee derives the simplified formula by explicitly assuming that “each district has exactly the same number of voters.”<sup>158</sup> Under this equal turnout assumption, for every district ( $i$ ) in the plan ( $D$ ), district-level turnout ( $T_i$ ) is equal to some constant turnout level ( $T^*$ ).

$$T_i = T^* \quad \forall i \in D$$

Thus, the simplified formula, by design, does not account for inter-district variation in voter turnout.<sup>159</sup> Thus far, I have used the same equal turnout assumption to derive the competitiveness gap. If a real-world election satisfies this assumption, the long-form formula for the efficiency gap, the simplified seats-votes formula, and the competitiveness gap equation will all compute precisely the same number.

$$T_i = T^* \quad \forall i \in D \rightarrow \Delta W_{xy} = S^* - 2V^* = \Delta C_{xy}$$

However, questions of plausibility and robustness arise whenever an assumption (like equal voter turnout) underlies a shorthand equation (like the simplified formula) for a measure (like the efficiency gap).<sup>160</sup> To address

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<sup>158</sup> McGhee, *supra* note 9, at 79 (Appendix B) (deriving the simplified formula) (first line of derivation explicitly assumes that “each district has exactly the same number of voters” so that “proportions can be substituted for raw votes in all of the formulas [and so] the total vote in each district becomes equal to 1.0,” and the sum of district vote totals “is simply the total number of districts in the electoral system.”); Stephanopoulos & McGhee, *supra* note 7, at 853 (citing McGhee’s derivation of the simplified formula in Appendix B of his 2014 article).

<sup>159</sup> McGhee, *supra* note 9, at 83 n.6 (“This necessarily assumes away differences in efficiency due to turnout...but turnout variation is still a worthy topic of study. In fact, future research could use Equation (2) [the long-form equation] instead of Equation (5) [the simplified formula] to explore the subject.”).

<sup>160</sup> Last term the Court clarified that a state *may* – but declined to address whether it *must* – comply with the one-person-one-vote principle by equalizing the number of *people* – as opposed to the number of *voters* – in each district. *Evenwel v. Abbott*, 578 U.S. \_\_\_\_ (2016). Thus, equal *total* population *might* be constitutionally required, but neither equal *voter* population nor equal *voter turnout* is constitutionally required. The scholarship presenting

these questions, I relax the equal voter turnout assumption and derive a more generalized simplified formula that expresses the efficiency gap in terms of statewide seat and vote margin but makes no *ex ante* assumption about inter-district variation in voter turnout.

a. *Turnout Gaps*

To explain the impact of turnout variation on the efficiency gap and the competitiveness gap, I first introduce the concepts of a turnout gap and a competitiveness-weighted turnout gap. When we relax the equal turnout assumption, we can still denote by  $T^*$  the *average* turnout across all districts:  $T^* = \frac{\sum_D T_i}{d}$ . But now each district may have turnout above or below (or equal to) average turnout. Let  $(\Delta T_i)$  denote the proportional difference between actual turnout in district(*i*) and average turnout over all districts:  $\Delta T_i = \frac{T_i - T^*}{T^*}$ .<sup>161</sup> Let  $(\Delta T_p)$  denote the average value of  $(\Delta T_i)$  over districts won by party P:  $\Delta T_p = \frac{\sum_{D_p} \Delta T_i}{d_p}$ .<sup>162</sup> Let  $(\Delta T_p^c)$  denote the competitiveness-weighted average of  $(\Delta T_i)$  over districts won by party P.  $\Delta T_p^c = \frac{\sum_{D_p} \Delta T_i c_i}{d_p}$ . Finally,

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the efficiency gap proposal, published pre-*Evenwel*, characterized the relevant equality conditions in a way that may generate confusion. Compare Stephanopoulos & McGhee *supra* note 7, at 853 (stating that the simplified formula assumes that “all districts are equal in *population* (which is constitutionally *required*)”) (emphasis supplied) with McGhee, *supra* note 9, at 68 (“[The long-form efficiency gap] reduces to [the simplified form]... [i]n the special case [where] districts are equal in *population* “); and *id.* at 83 n.6 (“Ignoring *turnout* differences in this way is legally *mandated* for redistricting in the United States.”) (emphasis supplied); and with *Evenwel v. Abbott*, 136 S.Ct. 1120, 1125, 1126-27 (2016) (declaring it “plainly permissible for jurisdictions to measure equalization by the *total* population of state and local legislative districts” even if this produces an electoral map that “measured by a *voter*-population baseline [exhibits a] maximum population deviation exceed[ing] 40%.”) (emphasis supplied). If the assumption used to derive the simplified formula were constitutionally required, it would necessarily be satisfied in real elections, and so questions of plausibility and robustness would be moot. Yet the relevant assumption is not equal *total population*, but equal *voter turnout*. This assumption is not constitutionally required; a constitutionally valid electoral map may exhibit small inter-district variation in *total population*, but large variation in the population of *eligible voters*, and even larger variation in actual *voter turnout* on election day. Thus, there is no guarantee that real elections will exhibit (or even approximate) equal voter turnout, and so questions of plausibility and robustness warrant attention.

<sup>161</sup>  $\Delta T_i$  is positive when district (*i*) has higher than average turnout; negative when district (*i*) has lower than average turnout; and zero when district(*i*) has average turnout.

<sup>162</sup> By design, with only two parties, the seat-share-weighted *sum* of  $(\Delta T_x)$  and  $(\Delta T_y)$  is zero.  $0 = \sum_D \Delta T_i = \sum_{D_x} \Delta T_i + \sum_{D_y} \Delta T_i = S_x \Delta T_x + S_y \Delta T_y$ . This makes intuitive sense because x-won districts will have above-average turnout only if y-won districts have below-average turnout (and vice versa).

define the **turnout gap**<sup>163</sup> as half the seat-share-weighted *difference* of  $(\Delta T_x)$  and  $(\Delta T_y)$  and define the **competitiveness-weighted turnout gap** as the seat-share-weighted *difference* of  $(\Delta T_x^c)$  and  $(\Delta T_y^c)$ .

$$\Delta T_{xy} = \frac{S_x \Delta T_x - S_y \Delta T_y}{2}$$

$$\Delta T_{xy}^c = S_x \Delta T_x^c - S_y \Delta T_y^c$$

Armed with these concepts, we can generalize both the seats-vote formula and the competitiveness gap in the case of turnout variation.

*b. Turnout-Generalized Formulae*

The turnout-generalized seats-votes formula is:<sup>164</sup>

$$\Delta W_{xy} = S^* - 2V^* + \Delta T_{xy}$$

The turnout-generalized competitiveness gap is:<sup>165</sup>

$$\Delta C_{xy} = \Delta W_{xy} + \Delta T_{xy}^c$$

$$\Delta C_{xy} = S^* - 2V^* + \Delta T_{xy} - \Delta T_{xy}^c$$

When we simultaneously generalize the equal voter turnout assumption and the definition and weight of surplus votes, the seats-votes formula is:

$$G = S - r^*V + \Delta T$$

$$\text{where } r^* = 1 + 2\gamma$$

This more generalized simplified formula is just like the prior one, except it now contains an additional term – variable  $\Delta T_{xy}$  - that precisely quantifies the effect of voter turnout on the efficiency gap. Note that a zero turnout gap does not necessarily entail equal average turnout in X-won and Y-won districts. This equality obtains only in the special case where each

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<sup>163</sup> Note that the turnout gap can be alternatively expressed as the sum of proportional turnout differences over x-won districts.

$$\Delta T_{xy} = \frac{S_x \Delta T_x - S_y \Delta T_y}{2} = S_x \Delta T_x = \sum_{D_x} \Delta T_i$$

<sup>164</sup> See Technical Appendix, Part 5.

<sup>165</sup> See Technical Appendix, Part 5.

party wins half the seats. In the more general case, the relative turnout of X-won and Y-won districts needed to achieve a zero turnout gap will depend on the relative seat shares. Formally, let ( $t$ ) denote the turnout ratio  $t = \frac{\Delta T_x}{\Delta T_y}$ , and ( $s$ ) the seat ratio ( $s = \frac{d_x}{d_y}$ ). Then the sign of the turnout gap ( $\Delta T_{xy} = 0$ ) depends on both the seat ratio ( $s$ ) and the turnout ratio ( $t$ ).

$$\begin{cases} \Delta T_{xy} > 0 & t > s \\ \Delta T_{xy} = 0 & t = s \\ \Delta T_{xy} < 0 & t < s \end{cases}$$

When the turnout ratio exceeds the seat ratio, the turnout gap is positive, and the simplified formula underestimates the efficiency gap; when the seat ratio exceeds the turnout ratio, the turnout gap is negative, and so the simplified formula overestimates the efficiency gap; when the turnout ratio equals the seat ratio, the turnout gap is zero, and so the simplified formula equals the efficiency gap.

An example of the first scenario is Indiana's 2014 congressional election. Out of 1,295,863 ballots cast, the Democrats wasted 379,150 votes, but the Republicans wasted only 268,782 votes, for an efficiency gap of 8.5% in Republicans favor according to the long form equation. With only 61% of the statewide vote share, the Republicans won seven of nine districts, for an efficiency gap of 5.3% according to the simplified formula. Why the discrepancy between the long-form equation (8.5%) and the simplified formula (5.3%) - between the departure from equal wasted votes and the departure from double-proportionality? Because there is a turnout gap. District turnout is 143,985 on average, but it ranges from a low of 108,330 in D-won district 7 to a high of 164,728 in R-won district, and turnout in R-won districts is 4.2% above average while turnout in D-won districts is 14.6% below average, for a turnout gap of 3.2%. The turnout gap is the difference between the long-form and simplified computations. Note that the long-form value (the departure from equal wasted votes) is greater than the short-form value (departure from double proportionality) because R-won districts have above-average turnout (for a positive turnout gap). Thus, the simplified formula underestimates the efficiency gap.

DISTRICT	TOTAL VOTES BY PARTY	
	R	D
1	51,000	86,579
2	85,583	55,590
3	102,889	39,771

4	94,998	47,056
5	105,277	49,756
6	102,187	45,509
7	46,887	61,443
8	103,344	61,384
9	101,594	55,016

In some cases, the turnout gap may be so small it has no meaningful effect on efficiency gap analysis and can thus be safely ignored. The 2012 Wisconsin Assembly election under the plan challenged in *Whitford* appears to be one such case. The turnout gap in that election appears to be less than 2%, so both the simplified formula and the long-form calculation produced similar above-threshold gaps.<sup>166</sup> For this reason, even though the *Whitford* majority regarded the long-form equation as “preferable” to the simplified formula, it was “untroubled” by the choice of computational technique, given that “both methods yield an historically large, pro-Republican [gap].”<sup>167</sup>

Consider the analysis of Dr. Kenneth Meyer. In this analysis, under the actual plan, of 2,844,676 votes cast, Democrats waste 877,445 votes, while Republicans waste only 544,893 votes, for an efficiency gap of 11.69% according to the long-form calculation.<sup>168</sup> In this plan, Republicans receive only 1,389,958 out of the total 2,844,676 votes cast, but win 57 out of 99 districts, for an efficiency gap of 9.85% according to the simplified formula.<sup>169</sup> The long-form value exceeds the short-form value because there

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<sup>166</sup> Specifically, Professor Mayer computed a gap of 11.69% using the long-form, while Professor Jackman computed a gap of 13% using the simplified formula. *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*51 (W.D. Wis. Nov. 21, 2016). This suggests that average turnout in Republican-won assembly districts was 1.31% higher than average turnout in Democrat-won assembly districts. However, in addition to employing different *computation* methods (long-form versus simplified formula), Professors Mayer and Jackman may have also employed different *imputation* methods to account for uncontested assembly races. Of 99 assembly races, 23 were won by unopposed Democrats and 4 were won by unopposed Republicans. Expert Report of Kenneth R. Mayer, *supra* note 140, at 40. See *supra* Part IV.B for a discussion of the efficiency gap’s sensitivity to alternative imputation strategies.

<sup>167</sup> *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*54 (W.D. Wis. Nov. 21, 2016). The *Whitford* majority also emphasized that defendant’s expert Nicholas Goedert “described the simplified method as ‘an appropriate and useful summary measure’,” *id.*, and both parties stipulated that the simplified formula’s “implied 2-to-1 votes-to-seats relationship reflects the ‘observed average seat/votes curve in historical U.S. congressional and legislative elections.’” *Id.* Finally, the *Whitford* majority cautioned that “[w]ere there record evidence indicating that [the simplified formula] did not correlate highly with both the [long-form] and electoral reality, we would have reason to doubt its validity.” *Id.*

<sup>168</sup> Meyer Table 10.

<sup>169</sup> Meyer Table 10.

is a turnout gap of 1.84% in favor of Republicans. But both values indicate above-threshold gaps in favor of Republicans.

But this is not always the case. In the 2014 congressional elections, Indiana had a turnout gap of 3.2%; Texas had a turnout gap of 8.9%.<sup>170</sup> When the turnout gap is large enough, the choice of computation method matters, presenting questions of robustness and correspondence.<sup>171</sup>

Finally, any factor that disproportionality decreases Democratic turnout will generally tend to generate a turnout gap in Republicans' favor. The reason is that Democrat-won districts have more Democrats, so a uniform decrease in Democratic turnout will have a larger impact on the turnout in Democrat-won districts than in Republican-won districts. This suggests that the turnout gap has a systematic tendency to increase whenever electoral rules have this differential partisan impact on turnout. Many believe, with good reason, that this is precisely the impact and intent of many recent electoral reforms.<sup>172</sup>

## 2. Voter Suppression and the Efficiency Gap

Because the efficiency gap is a measure of the relative number of wasted votes for each party, the only data points that it registers in assessing partisan fairness are ballots cast. In its most direct application, an analyst computes the efficiency gap produced by a given plan in a given election by inputting into the long-form equation (or simplified formula) the actual votes cast in that election by district and party (or the vote margin and seat margin produced by that election) after imputations for uncontested races. In a more advanced application, an analyst estimates the results of a hypothetical election using regression techniques on historical and contemporaneous data correlated with election outcomes, and then inputs those results into the relevant equation to compute the efficiency gap a given plan would likely produce in that hypothetical election.

The efficiency gap's singular focus on ballots cast means that it cannot detect any obstacles voters face in casting ballots, and cannot perceive voter suppression efforts that disproportionately impact supporters of one party. If voter suppression thwarts a voter's effort to cast a ballot, the efficiency gap perceives no problem; the measure simply assumes the relevant party has one ballot less support from the electorate. Unintentionally, this dynamic may reward, and thereby further incentivize, voter suppression efforts, because the suppression may make a gerrymander seem less like a gerrymander – it can hide gerrymanders from the efficiency gap. Suppressing

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<sup>170</sup> See Technical Appendix on file with author (computing these turnout gaps from officially reported election results.) Note that the latter number is larger than the proposed numeric threshold for state legislative plans.

<sup>171</sup> In Part V, *infra*, I discuss in detail the questions of robustness and correspondence raised by this and other issues.

<sup>172</sup> See *supra* notes 148-52 & accompanying text.

one party's statewide vote total can have the effect of reducing the overall gap.<sup>173</sup>

Voter suppression is a pressing matter of current national concern. Some recent electoral reforms, such as identification requirements for in-person voting, differentially burden Democratic-leaning voters. Other reforms eliminate or scale back pre-existing mechanisms that had differentially benefited Democratic-leaning voters, such as absentee voting or early voting. In the three years since the Supreme Court decided *Shelby County v. Holder*,<sup>174</sup> striking down the Voting Rights Act's coverage formula and thereby rendering inoperative its preclearance regime, fourteen states have adopted such electoral reforms.<sup>175</sup>

It is particularly problematic for a measure of partisan gerrymandering to be manipulable through voter suppression, because the two phenomenon may be correlated. Specifically, a Republican-dominated state is more likely to engage in voter suppression efforts, and more likely to engage in pro-Republican partisan gerrymandering, but voter suppression efforts may cause ballot-dependent measures like the efficiency gap to underestimate the pro-Republican advantage conferred by a partisan gerrymander.

Consider the two states currently defending against partisan gerrymandering claims based on efficiency gap approaches: Wisconsin and North Carolina. Each state exhibits the same pattern: after long periods of divided government, the Republican party won control of both the state legislature and the Governor's mansion, and promptly instituted both new electoral maps more favorable to Republicans than their predecessors and new electoral reforms with a predictably large and partisan effect on voter turnout. Take, for example, North Carolina, a state that was subject to preclearance prior to *Shelby County*, and a state where "African-American race is a better predictor for voting Democratic than party registration."<sup>176</sup> Once the Court struck down the coverage formula, the North Carolina legislature quickly enacted an omnibus electoral reform bill with five key provisions: (1) a voter identification requirement (for in-person but not mail-in voting) limited to forms of identification that whites tend to have and blacks tends to lack; (2) a reduction in early voting days; and elimination of

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<sup>173</sup> To be clear, I do not claim that this lack of sensitivity to voter suppression is a problem unique to the efficiency gap. It may be a feature of any measure of political gerrymandering that quantifies an ideal or acceptable relationship between votes won and seats won. Any definition that uses popular support as demonstrated by ballots cast to justify the number of seats won may incentivize voter suppression, because political actors seeking to demonstrate greater relative support can inflate the appearance of that support by preventing their competitor's supporters from voting.

<sup>174</sup> 570 U.S. \_\_\_\_ (2013).

<sup>175</sup> BRENNAN CENTER FOR JUSTICE, NEW VOTING RESTRICTIONS IN AMERICA, <https://www.brennancenter.org/new-voting-restrictions-america>.

<sup>176</sup> *N. Carolina State Conference of NAACP v. McCrory*, 831 F.3d 204, 225 (4th Cir. 2016).



(3) same-day registration; (4) out-of-precinct voting; and (5) pre-registration. The Fourth Circuit concluded that this omnibus bill constituted intentional racial discrimination, “target[ing] African Americans with almost surgical precision”<sup>177</sup> and thereby “[u]sing race as a proxy for party...to win an election.”<sup>178</sup> Wisconsin adopted a stringent voter identification law, which was subsequently softened after a federal court concluded it violated equal protection and the Voting Rights Act.<sup>179</sup>

What impact might these electoral reforms have on efficiency gap measures for Wisconsin and North Carolina plans? It is difficult to confidently estimate the impact of electoral reforms on voter turnout.<sup>180</sup> But consider the following facts: in the case of the 2012 Wisconsin assembly election, about 2.8 million ballots were cast; a federal judge concluded that about 300,000 registered voters in Wisconsin – 9% of registered voters – lacked the required photo identification;<sup>181</sup> and holding seat share constant, every 1% decrease in Democratic vote share decreases by 2% the Republican advantage conferred by an electoral map as estimated by the efficiency gap. If 14,000 fewer Democrats cast ballots, the pro-Republican efficiency gap would go down 1%. If about 50,000 fewer Democrats cast ballots, the pro-Republican efficiency gap would fall below the 8% numeric threshold. If 165,000 fewer Democrats cast ballots, the pro-Republican efficiency gap would be eliminated entirely.

The only way to avoid this perverse relationship between the turnout effects of partisan voter suppression and the measure of partisan gerrymandering is to explicitly account for partisan voter suppression in the measure. This can be accomplished by applying the efficiency gap measure to a hypothetical election result estimated in a way that controls for turnout-

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<sup>177</sup> *Id.* at 214.

<sup>178</sup> *Id.* at 222.

<sup>179</sup> See *Frank v. Walker*, 17 F. Supp. 3d 837 (E.D. Wis. 2014), *rev'd*, 768 F.3d 744 (7th Cir. 2014), *cert. denied*, 135 S. Ct. 1551 (2015); *Milwaukee Branch of NAACP v. Walker*, 851 N.W.2d 262 (Wis. 2014); *Frank v. Walker*, 819 F.3d 384 (7th Cir. 2016); Richard L. Hasen, *Softening Voter ID Laws Through Litigation: Is It Enough?*, 2016 WIS. L. REV. FORWARD 100, 110 (2016)

<sup>180</sup> Turnout may go down after an electoral reform, but this correlation does not tell us how much of the turnout effect was caused by the reform as opposed to other factors like reduced enthusiasm for races and candidates. We may be able to estimate the number of people potentially affected by an electoral reform – like the number of registered voters who presently lack required voter identification. But this does not tell us the number of voters actually affected – like the number of registered voters turned away on election day for lack of proper identification. Another complicating factor is that voter suppression efforts may affect turnout through information or misinformation. For example, an individual may actually be permitted by law to cast a ballot, but decline to try to vote based on the mistaken assumption that she would not be permitted to.

<sup>181</sup> *Frank v. Walker*, 17 F. Supp. 3d 837, 854 (E.D. Wis.), *rev'd*, 768 F.3d 744 (7th Cir. 2014)

reducing electoral reforms or practices. In essence, this technique would ask what efficiency gap a plan would produce without voter suppression, rather than what efficiency gap a plan would produce with voter suppression. The drawback is this makes the measure further reliant on hypotheticals and sensitive to modelling assumptions. But it would avoid rewarding and further incentivizing partisan voter suppression. At the very least, this approach warrants consideration under the circumstances present in Wisconsin and North Carolina, where the same lawmakers that enacted the challenged map enacted electoral reforms subsequently deemed unlawful that risk partisan voter suppression and a corresponding underestimation of the extent of advantage those maps conferred on the Republican party. More generally, if efficiency measures are to play a role in partisan gerrymandering claims, this problem warrants further consideration.

## V. DOCTRINAL IMPLICATIONS & CONCLUSIONS

By unpacking, reframing, and generalizing the efficiency gap measure, I have identified a series of related normative and technical concerns. I now provide concluding thoughts about the ways in which a legal test incorporating the efficiency gap measure should account for these concerns.

As described in Part II.C, the proponents<sup>182</sup> advance a legal test based on a constitutional ideal of equal wasted votes in which a sufficiently large and durable efficiency gap triggers a presumption of invalidity that can only be overcome with adequate justification.<sup>183</sup> The proponents suggest, without explicitly stating, that plans with below-threshold gaps enjoy an *irrebuttable* presumption of validity.

In this section, I build on my prior analysis to make two key recommendations with respect to the development of a legal standard. First, when calculating an individual plan's gap and – even more critically, when selecting the appropriate numeric threshold – courts should be cognizant of and seek to account for the problems of robustness and scope in the efficiency gap measure as proposed.

Second, courts should not adopt the efficiency gap measure as the sole definition of gerrymandering such that a below-threshold gap triggers an irrebuttable presumption of validity. A legal test based exclusively on the efficiency gap measure would suffer from correspondence problems: it would fail to capture important indicators of political manipulation that violate democratic norms of electoral competitiveness, seats-votes proportionality, and voter participation, and may thereby permit or even incentivize such manipulation.

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<sup>182</sup> I focus here on the legal framework proposed in scholarship, which differs slightly from subsequent proposals in the *Whitford* and *Rucho* cases.

<sup>183</sup> Stephanopoulos & McGhee, *supra* note 7, at 884.

A. *The Legal Test's Robustness and Scope*

This paper has analyzed how the efficiency gap measure depends on multiple methodological choices and electoral assumptions: the definition and weight of surplus votes; the imputation method for uncontested races; and, when using the simplified formula, the assumption of equal voter turnout across districts.<sup>184</sup> These choices and assumptions undermine the robustness and scope of the measure. Relaxing any of these assumptions has the potential to change the gap calculated; relaxing all of them may have a cumulative effect. Any legal test based on the efficiency gap measure must wrestle with the legal significance of these technical problems of robustness and scope. If, as proposed, the legal test compares the challenged plan's efficiency gap to some numeric threshold, both the challenged plan's gap and the numeric threshold itself pose challenges of robustness and scope.

At the level of a single plan, the simplifying assumptions and methodological choices will have legal significance if generalizing them (1) changes the sign of the gap and thereby toggles the assessment of which party is favored; (2) drives the gap above or below the numeric threshold, and thereby toggles the assessment of presumptive validity; or (3) changes the magnitude of the gap enough to influence the justification analysis (for example, a proffered justification may explain a gap of 10% but not a gap of 20%).

At the level of the numeric threshold, the simplifying assumptions and methodological choices will have legal significance if generalizing them would produce virtually any change in the numeric threshold – as the threshold will be used to indicate a presumptive gerrymander for all future plans. The numeric threshold is the benchmark for assessing the extent of an individual plan's efficiency gap. But the numeric threshold is itself based on a claim about the distribution of gaps associated with modern elections – and therefore susceptible to problems of robustness.

To illustrate these dynamics, let us consider how the proponents arrived at a numeric threshold of 8% for state legislative maps. The 2015 article used only the simplified formula to compute the efficiency gaps for a large number of congressional and state legislative elections between 1972 and 2012.<sup>185</sup> Based on the distributions, the authors proposed an 8% numeric threshold for state legislative maps.<sup>186</sup> In other words, gaps higher than 8% would trigger a presumption of the map's invalidity. They reasoned that a “gap of at least eight points placed a [state legislative] plan in the worst 12

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<sup>184</sup> Other related methodological choices – such as whether to look to statewide or district-level races when calculating the gap – also affect the measure's robustness.

<sup>185</sup> Stephanopoulos & McGhee, *supra* note 7, at 867-68.

<sup>186</sup> In the *Whitford* litigation, plaintiffs' expert Professor Jackman used only the simplified formula to compute the efficiency gaps for a similarly large number of elections. Based on these distributions, Jackman proposed a 7% numeric threshold. *Whitford v. Gill*, No. 15-CV-421-BBC, 2016 WL 6837229, at \*15 (W.D. Wis. Nov. 21, 2016).

percent of all plans in this period, also about 1.5 standard deviations from the mean.”<sup>187</sup>

However, this historically-derived 8% threshold depends on all of the methodological choices and simplifying assumptions discussed to this point. The method of imputing election results for uncontested elections is never fully explained; and we know that variation in imputation method, as described in Part IV.B, can sometimes produce significant changes in the gap. Because the gaps were calculated using the simplified formula rather than the long-form calculation, all differential turnout effects were ignored. And, of course, all gaps were calculated using proponents’ definition and weighting of surplus votes. The precise impact of all of these assumptions and methodological choices can only be quantified by re-calculating all of the historical gaps using more generalized formulas – an intensive project that is certainly worth undertaking, but which lies beyond the scope of this paper. Thus there is substantial uncertainty as to whether and to what degree the threshold of 8% would change if the underlying choices and assumptions were relaxed.<sup>188</sup>

It should be noted at this point that the proponents set a different threshold for congressional plans – two undeserved seats – rather than the 8% efficiency gap threshold for state legislative plans.<sup>189</sup> In arriving at this threshold, proponents added a new methodological choice to the mix: When calculating the historical undeserved seat share in congressional races, they omitted all election results from plans with fewer than eight districts.<sup>190</sup> This choice was justified on the ground that “redistricting in smaller states has

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<sup>187</sup> Stephanopoulos & McGhee, *supra* note 7, at 888-89.

<sup>188</sup> Even if the eight-percent threshold approximates the “right” answer because it turns out that the assumptions about electoral circumstances in fact do approximate real-world historical circumstances, we should be cautious about assuming that *future* elections will follow the same pattern and thus be susceptible to meaningful apples-to-apples comparisons to historical gaps. For example, it may be that average turnout differential has generally been near zero in past elections, but that it increases in future elections due to renewed voter suppression efforts. *See* Part IV.C.1, *supra*, describing recent voter suppression efforts after *Shelby County v. Holder*. Such voter suppression efforts may themselves be influenced by whether (and if so how) the Court adopts a legal standard for partisan gerrymandering based on an efficiency gap measure. *See* Part IV.C.1 (discussing how the efficiency gap may incentivize or ignore voter suppression). If so, then relying on the simplified formula to produce accurate results may be problematic.

<sup>189</sup> Stephanopoulos & McGhee, *supra* note 7, at 837. Note that the League of Women Voters in the *Rucho* litigation have not focused on the two-seat threshold but rather on a numeric efficiency gap threshold. It appears from plaintiffs’ complaint that, in calculating historical gaps across congressional plans, and evaluating the extent of North Carolina’s deviation from historical norms, states with low district-numerosity were also excluded from that historical baseline. *See* Common Cause v. Rucho, No. 1:16-CV-1026, 2017 WL 876307, at \*4 (M.D.N.C. Mar. 3, 2017) (“According to the League Amended Complaint, the Plan produced an efficiency gap of 19 percent in the 2016 election, which is “in approximately the worst 4 percent of the historical distribution, and the single worst score of all relevant congressional plans in the country in 2016.” *Id.* ¶¶ 61–62.”).

<sup>190</sup> Stephanopoulos & McGhee, *supra* note 7, at 868.

only a minor influence on the national balance of power.”<sup>191</sup> Yet twenty-two states (comprising more than one-fifth of all representatives) have more than one but fewer than eight congressional districts.<sup>192</sup> Removing these states from the full data set eliminates those data points from consideration and may significantly change the historical analysis.<sup>193</sup>

Special problems do arise in trying to apply the efficiency gap measure to plans with low district numerosity. If the threshold is set at two undeserved congressional seats then voters in many small states could never make political gerrymandering claims because there could never be two undeserved seats. Conversely, if the threshold is set as an efficiency gap of 8% or higher, maps with low district numerosity may be particularly likely to exhibit above-threshold gaps. For example, if a state, like Idaho, has only two congressional districts, there are only two possible values for seat margin: zero (each party wins one district) or fifty percent (one party wins both districts). Unless this state has the right vote share, it will necessarily have a large efficiency gap. We see, then, a problem of scope. The efficiency gap either works poorly or does not work at all in capturing gerrymandering dynamics in plans with low district numerosity, such as congressional maps for small states and electoral maps for local governing bodies like city councils and school boards.

In short, the efficiency gap calculation is not robust to methodological choice and input variation. Without acknowledging this robustness problem, we may unwittingly accept the accuracy of the numbers efficiency gap calculations produce even when they do not fully represent real-world circumstances – and these inaccuracies may have legal significance. I recommend utilizing the more generalized formulas presented in this paper to conduct a two-part data analysis procedure to assess robustness. First, we can re-do the historical analyses used to derive the numeric threshold, but this time we can explicitly account for different possible imputation methods and calculation methods and other methodological choices.<sup>194</sup> We can then be

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<sup>191</sup> *Id.*

<sup>192</sup> Twenty-two states were apportioned more than one but fewer than eight representatives in 2010. See U.S. Census Bureau, *Apportionment Population and Number of Representatives, by State: 2010 Census*, [https://www.census.gov/population/apportionment/data/2010\\_apportionment\\_results.html](https://www.census.gov/population/apportionment/data/2010_apportionment_results.html). In total, these states were allocated 91 representatives – 21% of the total number of representatives nationwide from states with more than one congressional district. *Id.* In order of delegation size, from smallest to largest, they are: Hawaii, Idaho, Maine, New Hampshire, Rhode Island, Nebraska, New Mexico, West Virginia, Arkansas, Iowa, Kansas, Mississippi, Nevada, Utah, Connecticut, Oklahoma, Oregon, Kentucky, Louisiana, Alabama, Colorado, and South Carolina.

<sup>193</sup> If one calculates the efficiency gap for all 2014 congressional maps, eight of the ten states whose congressional plans produce the highest gaps are small states that would have been excluded by the proponents.

<sup>194</sup> Similarly, scholars are increasingly conducting new types of efficiency gap analysis over large data sets. See, e.g., Krasno, *supra* note 16. When we start calculating a large number of gaps and making global claims about them, we should be careful to clearly

confident that the numeric threshold obtained is the correct standard against which to measure all other plans. Next, using the generalized formula and the robustness-checked threshold, we can assess the validity of other individual plans.<sup>195</sup>

*B. The Legal Test's Normative Correspondence*

By reframing and generalizing the efficiency gap measure, this article clarifies its relationship to other democratic norms that many would deem central to political gerrymandering: electoral competitiveness, seats-votes proportionality, and voter participation. These relationships, it turns out, are problematic.

The efficiency gap measure is at best agnostic and at worst antagonistic toward the goal of electoral competitiveness. Because the measure privileges the perspective of mapmakers serving party interests, it does not recognize harm to voters when elections are uncompetitive. The measure fails to recognize even extreme bipartisan gerrymanders, so long as the efficiency gap remains low. And a more competitive plan poses a greater risk of an above threshold gap in the face of vote swings.

With a simplifying assumption of equal voter turnout across districts, the efficiency gap is equivalent to double seats-votes proportionality. It idealizes districting plans in which a party supported by seventy-five percent of the electorate wins all of the seats. To the extent that normative intuitions support a system in which vote share and seat share should be roughly equal, the efficiency gap undermines that norm – and not only in the extreme idealized scenario.

Finally, the efficiency gap measure depends on turnout. As a result, like other measures justifying seats won in terms of ballots cast, it registers voter suppression simply as reduced support for the party suppressed. In some circumstances, suppressing the disfavored party's voters will actually serve to lower the gap. Thus the efficiency gap does not condemn – and may in fact encourage – voter suppression.

With these considerations in mind, I conclude by returning to the ultimate question: should the Court adopt the efficiency gap proposal? The answer is that the Court may use it as an indicative measure but should not adopt the efficiency gap as the exclusive definitional measure, so that the validity of a plan turns primarily on whether it produces a sufficiently high efficiency gap. The other doctrinal elements of the proposed legal test – sensitivity analysis, intent, and justification – only partially address the false-

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identify all of the relevant methodological choices underlying the calculations so we can make apples to apples comparisons. For example, we know that Jackman used the simplified formula. It is not apparent what imputation method he used. It is unclear whether Krasno et al. used the simplified formula and what their imputation method was.

<sup>195</sup> Actually conducting this data analysis is a massive project and lies beyond the scope of this paper, but my hope is that this article may help to guide that future data analysis.

positive problem. In pursuit of competitiveness or proportionality, a mapmaker may devise a districting plan that produces an above-threshold gap, but still avoid invalidation unless that gap is durable, unjustified, and a result of discriminatory intent. But these doctrinal tools are imperfect; they may not always work, and even when they do, they avoid invalidation, not litigation. In contrast, a *below*-threshold efficiency gap triggers a presumption of *validity*, and the other doctrinal elements do nothing to address this false-negative problem. Since the measure is definitional and not merely indicative, the presumption appears to be *irrebuttable*. Were the Court to embrace this approach, any plan that wastes (roughly) equal votes would enjoy absolute immunity from judicial scrutiny, no matter how severely it subverted other democratic norms like competitiveness or seats-votes proportionality. In this way, the efficiency gap proposal would provide mapmakers a powerful incentive to draw uncompetitive plans that accord the majority a “double-proportionality” seat bonus. Such plans can be sensibly called gerrymanders if that term implicates norms of competitiveness and seats-votes proportionality. Yet those are the only plans the efficiency gap proposal is certain to approve, and thus the plans the proposal encourages mapmakers to design.

Since electoral districting implicates – and gerrymandering threatens – multiple democratic norms, it is unsurprising that a single measure fails to capture them all. Indeed, it may be that no one measure can satisfactorily reduce to a single number the multiple democratic norms at stake. I do not fault the efficiency gap measure for failing to perform an impossible task. And I recognize that the efficiency gap does measure one significant democratic norm, which may powerfully capture, and thereby help curb, certain partisan gerrymanders. For this reason, the efficiency gap is a helpful indicative measure that courts can and should consider when analyzing claims of partisan gerrymandering – so long as that consideration recognizes its technical and normative limits.