PRICE CAPS IN MULTI-PRICE MARKETS

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I. Introduction

The Credit Card Accountability, Responsibility and Disclosure Act of 2009 (the CARD Act), and its implementing regulations, imposed restrictions on certain dimensions of the credit card price. In particular, late fees were subjected to a de facto price cap. Other fees and interest rates were also curtailed. A few years later, the Dodd-Frank Act, and its implementing regulations, restricted the permissible magnitude of prepayment penalties in mortgage contracts.

In both of these examples, Congress, responding to concern about an excessively high price, resolved to cap the suspect price. Congress did not fully account, however, for the possibility of unintended consequences. In particular, credit cards and mortgages are both multi-dimensional products with multi-dimensional prices. When the law caps one price dimension, we cannot assume that other price dimensions will remain unchanged. If sellers react to the new law by increasing other prices, then it is no longer clear that the law will achieve its stated purpose.

Will the price cap increase social welfare? Will it make consumers better off? To answer these questions we need to first understand the forces driving the pre-cap pricing structure. If prices were efficient, designed to provide optimal incentives, then a price cap will likely reduce social welfare and hurt consumers. These distortions would be exacerbated in a multi-price market, where a price cap on one dimension can lead to adjustment away from the efficient level also on other price dimensions.

If, on the other hand, prices were not efficiency-based, but rather designed to exploit consumer biases, then legal intervention may increase welfare and help consumers. Indeed, if a certain price was set above the efficient level because this price dimension was non-salient to consumers, then capping this price could prove helpful, even if it results in a corresponding increase in other prices dimensions. Specifically, by reducing a non-salient price and increasing a salient price, the law would help consumers to more fully appreciate the total price of the product that they are purchasing.

The implications of a price cap also depend on market structure. Most importantly, the effect of a price cap on other, unregulated price dimensions depends on the seller's market power. In a competitive market, a reduction in one price will generally force the seller to increase another price, if the seller is to cover her overall costs. Not so in a monopolistic market. In the pre-cap world, the monopolist may have decided to increase the non-salient price, since the increase did not significantly reduce demand for the monopolist's product. Increasing a salient price, in response to a cap on the non-salient price, would cost more in terms of reduced demand, and the monopolist may well decide not to increase the unregulated price. The price cap would thus cut into the monopolists profits and increase consumer surplus.

Related Literature. Markets with multi-dimensional products, and multi-dimensional prices, have been studied in the I/O literature. Products with an aftermarket – for parts or service – provide a key example. See Farrell and Klemperer (2007) and Farrell (2008). In the behavioral I/O literature, several papers study multi-dimensional pricing. See, e.g., DellaVigna and Malmendier (2004), Gabaix and Laibson (2006), Heidhues, Koszegi and Murooka (2012).

These papers do not consider price caps.[1] The exception is Heidhues and Koszegi (2010). H&K focus on credit contracts, but their model could be generalized. Importantly, H&K study only the perfect competition case and thus do not identify the effects of market structure on the welfare implications of the price cap.

This paper builds on and extends the analysis in Bar-Gill & Bubb (2012). B&B study a simple, linear model where each price, in the two-dimensional pricing scheme, is incurred exactly once. The implication is that prices do not have incentive effects, beyond the purchase decision. The current paper relaxes some of the simplifying assumptions in B&B, generalizing and refining the results of that paper and deriving additional results.

[1] Gabaix and Laibson (2006) mention price caps in one short paragraph, with no analysis.

II. Model

A. Framework of Analysis

1. Basic Setup

Assume a two-dimensional product (X, Y). The consumer chooses how much to consume on each dimension, i.e., the consumer chooses consumption levels (x, y). Assume initially that X is a binary dimension, i.e., $x \in \{0,1\}$, with $x = 1$ representing a decision to purchase the product and $x = 0$ representing a decision not to purchase the product. If the consumer decided to purchase the product, she must then decide how intensely to use the product on the Y dimension, where $y \in R^{+}$.

The seller's cost of providing the product is separable, with an independent per-unit cost for each dimension of the product. There is a fixed cost, c_x, of serving any consumer who choses to purchase the product, and a per-unit cost, c_y, for each unit of use on dimension Y. The seller's total cost, for a consumer who decided to purchase the product, is: $C(c_x, c_y) = c_x + yc_y$.

The (gross) value of the product to the consumer is: $v + u(y)$, where v is a base-value that is distributed among consumers according to the CDF $F(v)$, and $u(y)$ is a use value that varies with use levels on the Y dimension but in a manner common to all consumers. (Explore the case where the use value, including misperceptions about the use value, vary across consumers.) I assume that $u'(y) > 0$ and $u''(y) < 0$.

2. The Seller's Decisions

The seller sets a two-dimensional price, which is comprised of a per-unit price for each dimension of the product. The per-unit prices are: p_x and p_y. The total price is: $P(p_x, p_y) = p_x + yp_y$. The prices that the seller sets depend, among other things, on market structure. I will consider two different assumptions about the structure of the market: perfect competition and monopoly.

The seller's profit function is:

$$\Pi\left(p_x, p_y\right) = \left(P\left(p_x, p_y\right) - C\left(c_x, c_y\right)\right) \cdot \left[1 - F\left(\tilde{v}\left(p_x, p_y\right)\right)\right]$$

where $\tilde{v}\left(p_x, p_y\right)$ represents a threshold base-value, such that consumers with base-values $v > \tilde{v}\left(p_x, p_y\right)$ purchase the product. This threshold value is derived below.

In a perfectly competitive market, prices will be set to maximize the (net) value of the product, as perceived by consumers, subject to a zero-profit constraint: $\Pi\left(p_x, p_y\right) = 0$. In a monopolistic market, prices will be set to maximize $\Pi\left(p_x, p_y\right)$.

3. The Consumer's Decisions

The consumer makes two decisions: (1) whether to purchase the product, and (2) how intensely to use a product that is purchased. These decisions are best described in reverse order:

Use decision. A consumer who has decided to purchase the product will choose a use level, y, that solves: $\max_y \langle u(y) - y p_y \rangle$. The FOC is: $u'(y) = p_y$, which implicitly defines the optimal use level as a function of the per-unit price, p_y: $y = y\left(p_y\right)$.

Purchase decision. The consumer will purchase the product if the perceived (net) value of the product is positive. Ex ante, when deciding whether or not to purchase the product, the consumer believes that her use value will be $\delta u(y)$, where $\delta \in [0,1]$. The benchmark case, where the consumer does not suffer from any misperception, is captured by $\delta = 1$. A smaller δ represents varying degrees of underestimation. Ex post, when the actual use decision is made, the consumer learns her true use value, $u(y)$, and sets the use level, y, accordingly. (Explore the alternative assumption that the misperception persists at the ex post stage.)

From an ex ante perspective, the consumer thinks that she will choose a use level, y, that solves $\max_y\langle\delta u(y) - yp_y\rangle$. The FOC is: $\delta u'(y) = p_y$, which implicitly defines the anticipated use level as a function of the per-unit price, p_y, and the misperception parameter, δ: $\hat{y} = \hat{y}(p_y; \delta)$.

The perceived value of the product to the consumer, net of the price, is:

$$V(p_x, p_y; \delta) = v + \delta u(\hat{y}(p_y; \delta)) - (p_x + \hat{y}(p_y; \delta)p_y).$$

A consumer with $V > 0$, or $v > \tilde{v}(p_x, p_y; \delta) \equiv (p_x + \hat{y}(p_y, \delta)p_y) - \delta u(\hat{y}(p_y, \delta))$ will purchase the product. Assuming a unit mass of consumers, the demand for the product is: $D(p_x, p_y; \delta) = 1 - F\left(\tilde{v}(p_x, p_y)\right)$.

4. The Social Optimum

A consumer who has decided to purchase the product should choose a use level, y, that solves: $\max_y\langle u(y) - yc_y\rangle$. The FOC is: $u'(y^*) = c_y$, which implicitly defines the optimal use level as a function of the per-unit price, p_y: $y^* = y^*(c_y)$.

A consumer should choose to purchase the product *iff* $v > \tilde{v}^*(c_x, c_y) \equiv (c_x + y^*(c_y) \cdot c_y) - u(y^*(c_y))$. The product should be purchased by the following number of consumers: $D^*(c_x, c_y) = 1 - F\left(\tilde{v}^*(c_x, c_y)\right)$.

The social welfare function is:

$$W = \int_0^{\tilde{v}^*(c_x, c_y)} \left[v + u(y) - (c_x + yc_y)\right] f(v)dv$$

5. The Law

I study the effects of a rule that restricts the permissible magnitude of p_y. Specifically, I consider a price cap, $\bar{p}_y$, that adds a "legal constraint" $p_y \leq \bar{p}_y$ to the seller's optimization problem. The question is under what conditions will such a law increase social welfare and under what conditions will it decrease social welfare.

The answer to this question depends on market structure. I begin with a competitive market and then proceed to consider a monopolistic market.

B. Competition

I first analyze a perfectly competitive market. In a competitive market, sellers set prices to maximize the perceived (net) value of the product in the eyes of consumers, subject to a break-even constraint. Formally, the seller solves the following maximization problem:

$$\max_{p_x,p_y} V(p_x, p_y; \delta) \text{ s.t. } \Pi(p_x, p_y) = 0$$

Substituting the expressions derived for $V(p_x, p_y; \delta)$ and $\Pi(p_x, p_y)$, the maximization problem becomes:

$$\max_{p_x,p_y} \langle v + \delta u(\hat{y}(p_y; \delta)) - (p_x + \hat{y}(p_y; \delta) p_y) \rangle \text{ s.t. } p_x + y(p_y) \cdot p_y = c_x + y(p_y) c_y$$

Or:

$$\max_{p_y} \langle v + \delta u(\hat{y}(p_y; \delta)) - (c_x + y(p_y) c_y - y(p_y) \cdot p_y + \hat{y}(p_y; \delta) \cdot p_y) \rangle$$

Applying the Envelope Theorem, the FOC simplifies to:

$$(1) \quad p_y = c_y + [\hat{y}(p_y; \delta) - y(p_y)] / \tfrac{dy(p_y)}{dp_y}$$

from which the price, p_y, can be derived. The price, p_x, can be derived from the zero profit constraint, $\Pi(p_x, p_y) = 0$:

$$(2)\ \ p_x = c_x - y(p_y) \cdot (p_y - c_y)$$

Lemma 1 defines the equilibrium prices in a competitive market.

Lemma 1: In a competitive market –

(a) Without misperception, i.e., when $\delta = 1$*:*

i. *The per-use price,* p_y*, is:* $p_y = c_y$.

ii. *The base price,* p_x*, is:* $p_x = c_x$.

iii. *The socially optimal welfare level is obtained.*

(b) When use value is underestimated, i.e., when $\delta < 1$*:*

i. *The per-use price,* p_y*, is:* $p_y > c_y$.

ii. *The base price,* p_x*, is:* $p_x < c_x$.

iii. *Social welfare is reduced through insufficiently low demand and inadequate use of purchased products.*

Proof: See Appendix.

Remark:

(a) This part of the lemma replicates standard results regarding the efficiency properties of perfect competition, under the (standard) assumption that consumers do not suffer from any misperception.

(b) This part of the lemma deals with the effects of consumer misperception.

i. If consumers underestimate use-values, they will also underestimate use levels and the importance of the per-use price, p_y. Sellers, responding to this misperception, will increase p_y above the efficient level.

ii. The zero-profit condition implies that an increase in one price, p_y, must be accompanied by a decrease in another price, p_x.

iii. These deviations from efficient pricing reduce social welfare. A per-use price that is set too high leads to inefficiently low use levels. Given the low use levels, and the correspondingly low use values, demand for the product should be at a second-best level that is lower than the first-best level of demand. But consumer misperception reduces demand even below this second-best level. Note that misperception would reduce demand below the first-best level, even if prices were efficient, i.e., if $p_x = c_x$ and $p_y = c_y$. The price distortions, however, exacerbate the adverse effects of the misperception.

We can now study the effects of imposing a price cap $\bar{p}_y$, namely of adding a legal constraint $p_y \leq \bar{p}_y$. We first note that, without misperception, a price cap can only reduce social welfare. Specifically, a price cap $\bar{p}_y \geq c_y$ will have no effect, since the constraint will not bind; and a price cap $\bar{p}_y < c_y$ will distort prices – reducing p_y and increasing p_x - resulting in a welfare loss. The reduced p_y leads to excessive use-levels. Moreover, the deviation from optimal pricing reduces the (net) value of the product and, thus, demand for the product. To see this, take the derivative of the (net) value $V(p_y) = v + u(y(p_y)) - (c_x + y(p_y)c_y)$ with respect to p_y:

$$\frac{dV(p_y)}{dp_y} = \left(u'\left(y(p_y)\right) - c_y\right)\frac{dy(p_y)}{dp_y} = (p_y - c_y)\frac{dy(p_y)}{dp_y}$$

Since $p_y = \bar{p}_y < c_y$, this derivate is positive in the relevant range, which means that forcing a lower p_y would reduce the (net) value of the product.

These results are stated in the following proposition.

Proposition 1: In a competitive market without consumer misperception, i.e., when $\boldsymbol{\delta = 1}$, a price cap, $\boldsymbol{\overline{p}_y}$, can only reduce social welfare.

Things are more complicated, and more interesting, when consumers underestimate the use value, i.e., when $\delta < 1$. Without legal intervention, consumer misperception results in an excessively high p_y and an inadequately low p_x (see Lemma 1). A price cap, $\bar{p}_y$, can reduce these distortions, as long as the cap is not set too low. These results are summarized in Proposition 2.

Proposition 2: In a competitive market with consumer misperception, i.e., when $\boldsymbol{\delta} < 1$, a price cap, $\boldsymbol{\overline{p}_y}$, can either increase or decrease social welfare. In particular –

(a) A mild, though still binding, legal constraint, $\boldsymbol{\overline{p}_y \geq c_y}$,

- **i. Reduces the per-use price, $\boldsymbol{p_y}$.**
- **ii. Either increases or decreases the base price, $\boldsymbol{p_x}$.**
- **iii. Either increases or decreases social welfare.**

(b) A strict legal constraint, $\boldsymbol{\overline{p}_y < c_y}$, can either increase or decrease social welfare.

Proof: See Appendix.

Remark:

(a) A mild, yet binding legal constraint brings the per-use price closer to the first-best price. This increases both the efficiency of the use-level decision and the total (net) value of the product.

The reduced p_y can either decrease seller revenue on the Y dimension or increase these revenues if the resulting rise in use level more than offsets the lower per-use price. If revenues on the Y dimension decrease, then the seller will raise p_x; if they increase, the seller will lower p_x. If both p_y and p_x go down, then demand for the product will increase and social welfare would be unambiguously higher. If, however, p_x does up, then demand for the product might decrease, resulting in a welfare loss.

(b) A strict constraint reduces the per-use price below the first-best price. This can either increase or decrease the efficiency of the use-level decision, as we move from an inadequately low use level (induced by $p_y > c_y$) to an excessively high use level (induced by $p_y = \bar{p}_y < c_y$). Similarly, the total (net) value of the product can either increase or decrease.

C. Monopoly

A monopolistic seller maximizes its profit function:

$$\Pi(p_x, p_y) = \pi(p_x, p_y) \cdot \left[1 - F\left(\tilde{v}(p_x, p_y)\right)\right]$$

where: $\pi(p_x, p_y) = P(p_x, p_y) - C(c_x, c_y) = p_x - c_x + y(p_y)(p_y - c_y)$

$$\tilde{v}(p_x, p_y; \delta) = \left(p_x + \hat{y}(p_y, \delta)p_y\right) - \delta u(\hat{y}(p_y, \delta))$$

[TBA]

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Appendix

Proof of Lemma 1

Consider Equation (1) and focus on the expression $\left[\hat{y}\left(p_y;\delta\right)-y\left(p_y\right)\right]/\frac{dy(p_y)}{dp_y}$. This expression equals zero for $\delta=1$, which proves Part (a.i). To prove Part (b.i), we show that the expression is positive for $\delta<1$. First, taking the derivative of the FOC that determines the use level, $u'(y)=p_y$, with respect to p_y, we obtain: $\frac{dy(p_y)}{dp_y}=\frac{1}{u''}<0$. It remains to show that $\hat{y}\left(p_y;\delta\right)-y\left(p_y\right)<0$. We note that $\hat{y}\left(p_y;\delta=1\right)=y\left(p_y\right)$, and show that $\frac{d\hat{y}(p_y;\delta)}{d\delta}>0$. This last inequality follows if we take the derivative of the FOC that determines the perceived use level, $\delta u'(\hat{y})=p_y$, with respect to δ: $\frac{d\hat{y}(p_y;\delta)}{d\delta}=-\frac{p_y}{\delta^2 u''}>0$.

Next consider Equation (2). Part (a.ii) follows from Part (a.i) and Equation (2). Part (b.ii) follows from Part (b.i) and Equation (2).

Finally, consider welfare effects. Part (a.iii) follows immediately from the optimality of the prices, as established in Parts (a.1) and (a.ii). For Part (b.iii), note first that excessive per-use prices distort use decisions, thus reducing social welfare. Next consider the consumer's purchase decisions: Purchase decisions are based on perceived (net) value:

$$V\left(p_x,p_y;\delta\right)=v+\delta u(\hat{y}\left(p_y;\delta\right))-\left(p_x+\hat{y}\left(p_y;\delta\right)p_y\right).$$

Given the low use levels caused by the excessive per-use price, demand for the product *should* be lower than the first-best level of demand. This second-best demand is determined by the following (net) value:

$$v + u(y(p_y;\delta)) - (p_x + y(p_y;\delta)p_y)$$

We show that this (net) value is greater than the perceived (net) value for all $\delta < 1$. Consider the difference between the perceived (net) value and the second-best (net) value:

$$\Delta V(p_x, p_y; \delta) = \delta u\left(\hat{y}(p_y;\delta)\right) - u\left(y(p_y)\right) - \left(\hat{y}(p_y;\delta) - y(p_y)\right)p_y$$

This difference is zero for $\delta = 1$. We show that the difference is increasing in δ, which implies $\Delta V(p_x, p_y; \delta) < 0$ for all $\delta < 1$. Taking the derivative, with respect to δ, and simplifying (using the Envelope Theorem), we obtain:

$$\frac{d\Delta V(p_x, p_y; \delta)}{d\delta} = u\left(\hat{y}(p_y;\delta)\right) - \frac{dp_y}{d\delta}\left(y(p_y) - \hat{y}(p_y;\delta)\right)$$

We first show that $\frac{dp_y}{d\delta} < 0$: We have seen that $\frac{d\hat{y}(p_y;\delta)}{d\delta} = -\frac{p_y}{\delta^2 u''} > 0$. Also, note that $\frac{d\hat{y}(p_y)}{dp_y} = \frac{1}{\delta u''} < 0$ (take the derivative of the FOC that determines the perceived use level, $\delta u'(\hat{y}) = p_y$, with respect to p_y). Taken together, these two inequalities imply $\frac{dp_y}{d\delta} < 0$. Since $\hat{y}(p_y;\delta) - y(p_y) < 0$ (see above), we have: $d\Delta V(p_x, p_y; \delta)/d\delta > 0$.

QED

Proof of Proposition 2

(a) Assuming that the legal constraint is binding, the per-use price will be $p_y = \bar{p}_y$, which is closer to the first-best price, $p_y = c_y$. This proves Part (a.i).

The base price, p_x, is determined by the zero-profit constraint:

$$p_x + y(\bar{p}_y) \cdot \bar{p}_y = c_x + y(\bar{p}_y)c_y$$

Or:

$$p_x = c_x - y(\bar{p}_y)(\bar{p}_y - c_y)$$

This price can be either higher or lower than the unconstrained price. This proves Part (a.ii).

We next prove Part (a.iii). First, since the per-use price is closer to the first-best price, distortions of use-level decisions, caused by consumer misperception, are reduced. Second, the (net) value of the product increases: Take the derivative of the (net) value $V(p_y) = v + u(y(p_y)) - (c_x + y(p_y)c_y)$ with respect to p_y:

$$\frac{dV(p_y)}{dp_y} = \left(u'\left(y(p_y)\right) - c_y\right)\frac{dy(p_y)}{dp_y} = (p_y - c_y)\frac{dy(p_y)}{dp_y}$$

Since $p_y = \bar{p}_y > c_y$, this derivate is negative in the relevant range, which means that forcing a lower p_y would increase the (net) value of the product.

But, given consumer misperception, an increase in the (net) value of the product does not necessarily mean that demand will increase. Demand is a function of the perceived (net) value of the product:

$$V(p_y) = v + \delta u(\hat{y}(p_y)) - (p_x(p_y) + \hat{y}(p_y)p_y)$$

Taking the derivative with respect to p_y, we obtain:

$$-\frac{dp_x}{dp_y} - \hat{y}(p_y)$$

Substituting $\frac{dp_x}{dp_y} = -\frac{dy}{dp_y}(p_y - c_y) - y(p_y)$, which can be derived from Equation (2), we obtain:

$$-\frac{dp_x}{dp_y} - \hat{y}(p_y) = \frac{dy}{dp_y}(p_y - c_y) + y(p_y) - \hat{y}(p_y)$$

which can be either positive or negative.

(b) TBA