

Taxation and Risk-Taking with Multiple Tax Rates

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Abstract

This paper extends the Domar and Musgrave results concerning the effect of an income tax on risk taking to the case where different tax rates are applied to different types of assets. Although the results depend on exactly how the differential tax rates are imposed, as a general matter, an income tax with differential rates can be seen as a tax only on the risk-free rate of return and an ex ante subsidy for purchasing the lower-taxed assets. There are implications for measuring deadweight loss from differential taxation and for spending resources on accurately measuring capital income.

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Since Domar and Musgrave (1944) and subsequent literature, it has been generally known that taxpayers can make portfolio adjustments that effectively eliminate the portion of an income tax nominally imposed on the risky return to investments.¹ It follows that a pure capital income tax is equivalent (both *ex ante* and *ex post*) to a tax on the riskless return on all assets. The most widely discussed implication of this conclusion is that it means there is little difference between an income tax and a consumption tax, and no difference at all between an income tax and a wealth tax.² In addition, the conclusion has been used to design taxes that are equivalent to an income tax but easier to administer.³

The proofs of this proposition are now fairly general. They place minimal restrictions on the utility function of individuals and they apply in a general equilibrium context. Most proofs are stated in terms of a choice of just two assets, a risky and a risk-free asset, but recent models are easily extended to multiple assets. Some models place few, if any, restrictions on the use of government revenues.

An important detail left out of the current models, however, is the possibility of different tax rates on different types of capital.⁴ Differential tax rates arise both because of purposeful deviations in tax rates, such as the corporate tax or special subsidies, and because of the difficulty in measuring capital income. They are an endemic feature of income taxes. This paper shows how the Domar/Musgrave results extend to this case. In particular, when there are differential tax rates on capital, taxpayers, as a general matter, can eliminate the tax on the risky return to capital in exchange for a fixed subsidy for purchasing the lower-taxed asset. The subsidy is based on the difference in the tax-exclusive tax rates on the *risk-free* rate of return for different types of capital. The exact nature of the portfolio adjustments, the costs of the adjustment, and even the ability to make such an adjustment, however, relate to the details of how differential tax rates are imposed.

There are a number of important policy implications of this conclusion. First, the conclusion affects the size of the deadweight loss from differential capital income taxation.

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¹Contributions in economics journals include Atkinson and Stiglitz (1980), Bulow and Summers (1984), Feldstein (1969), Gordon (1985) Kaplow (1995), Mossin (1968), Sandmo (1977), Stiglitz (1969), and Tobin (1958). In the legal literature, see Bankman and Griffith (1992) and Warren (1980, 1996).

²See, for example, Bankman and Fried (1998), Gentry and Hubbard (1997), and Kaplow (1995).

³See Auerbach (1991).

⁴Important exceptions are Gordon (1985) and Sandmo (1977)

Differential tax rates cause individuals to shift investment patterns away from the first best, resulting in deadweight loss. The extent to which an income tax actually taxes the return from capital will affect the extent to which individuals will alter their behavior. Prior estimates almost uniformly assume that differential tax rates apply to the full return from capital, essentially ignoring risk.⁵ To the extent the Domar/Musgrave results extend to the case of multiple rates, these estimates must be revised. The costs of the corporate tax or special preferences may be different than previously thought.

Second and closely related, measurement of income from capital is difficult. For example, measurement of capital income frequently involves estimating schedules of future asset depreciation, which cannot be done with any accuracy at a reasonable cost. Policymakers must decide how much to spend on measurement costs. To the extent the results concerning the riskless return apply, accuracy may be less valuable than one might think because inaccuracies affect only the riskless return.

To illustrate both points with a grossly simplified example, suppose that the risk-free rate of return is 3% and there are two risky capital assets, each with a return of 10%. Finally, suppose that one of the assets bears a tax of 33% and the other a tax at twice that rate, or 66%, with the difference arising either from a measurement error or an explicit subsidy. If these taxes are thought to fall on the full rate of return, the after-tax return on the low-tax asset would be 6.7% while the after-tax return on the high-tax asset would be 3.3%. This differential would potentially lead to a large deadweight loss. But if the tax applies only to the risk-free rate of return, the low-tax asset would face a 9% rate of return while the high-tax asset would face only an 8% rate of return, a much smaller difference. The deadweight loss from the tax difference would be relatively small, and spending resources to move the effective tax rates closer might not be worthwhile.

The models used here are based on the model found in Kaplow (1995). Kaplow's model is a simple, intuitive, yet general equilibrium model that can easily be modified to model multiple tax rates. The closest results to the results in this paper, however, are those of Gordon (1985). Gordon's basic model is of the corporate tax but in an extension, he discusses the possibility of variations in corporate tax rates (and equivalently, the possibility of non-corporate investment). His conclusions are similar to but not exactly the same as the conclusions here. Unlike Gordon's model, the results here are not dependent on lump sum transfers by the government or any assumptions about corporate behavior or finance. Another related paper, discussed below, is Bulow and Summers (1984), which argues that the Domar/Musgrave argument does not apply to most of the return on depreciable assets in part because different components of the return are taxed at different rates. The models here can

⁵For recent examples, see Mackie (2002), Jorgenson and Yun (2001), Treasury (2000).

be modified to cover and extend the Bulow and Summers story, but I will argue that it is not very realistic.

Part I gives the background, restating portions of Kaplow's model. Part II extends this model to the case of multiple tax rates. Part III discusses implications, and Part IV concludes.

I. The Basic Model

This section presents the basic model found in Kaplow (1995). The model provides good background on the nature of the problem and the case of multiple tax rates builds easily and directly off this model.

Assume that a representative individual has period 0 wage (or other) income of 1 all of which is invested and consumed in period 1. The individual is assumed to have a concave utility function of period 1 wealth (and period 1 government revenue). The individual has two choices of investments: a risk-free asset with return of r and a risky asset with a risky return of x (which, depending on the state of the world, can turn out to be either positive or negative).⁶ The individual chooses a , the portion invested in the risky asset. The amount invested in the risky asset, a , is not restricted to being greater than zero or less than one (i.e., the individual can short either the risky or risk-free asset). In the absence of taxes, the individual's wealth in period 1 is:

$$W = a(1+x) + (1-a)(1+r) \quad (1)$$

Suppose that a tax at rate t nominally applies to the entire return. We want to show that this is equivalent to a regime that applies the tax rate t to just the risk-free return. Following Kaplow (1995), two tax regimes will be treated as equivalent only if they look identical in all respects to both the government and all taxpayers. In particular, two tax regimes are equivalent if and only if (i) taxpayers have the same after-tax wealth in both regimes regardless of the state of nature; (ii) total (net) investment in each asset is the same in period 0; and (iii) the government has the same revenue in each regime, regardless of the state of nature.

The individual's wealth with the tax on the entire return is:

$$W_t = W - ax - (1-a)rt \quad (2)$$

⁶Although not specified, these assets can generally be thought of as liquid securities. See section II below for discussion of assets whose tax patterns are more complex.

with the second two terms representing the tax on the risky asset and on the risk-free asset respectively. We can write $ax_t = a(x-r)t + art$, and substitute, getting:

$$W_t = W - rt - a(x-r)t \quad (3)$$

A tax just on the risk free return would leave the taxpayer with

$$W_{rf} = W - rt \quad (4)$$

The additional tax due when a tax is imposed on the full return as opposed to just the risk-free rate of return is the difference between (3) and (4), or $a(x-r)t$. This additional tax can be costlessly financed by purchasing $at/(1-t)$ of the risky asset and selling (short if necessary) $at/(1-t)$ of the risk-free asset. Regardless of the state of nature in period 1, the individual has exactly enough to pay his tax if he holds this combination of positions. Because holding this combination of positions is costless, a tax on the full return looks exactly the same to the individual as a tax on the risk-free rate of return. Similarly, a tax on the risk-free rate of return looks the same to the individual as a tax on the full return. Therefore, the individual *can* have exactly the same amount in each regime regardless of the state of nature. Moreover, the individual *will* have the same amount in each regime as well, because the opportunity set is the same under each regime.

To fill this out a little bit more, which will be useful for the more complicated cases, we can explicitly solve for the portfolio adjustments needed to finance the additional tax liability from a tax on the full return instead of a tax on the risk-free return. The portfolio adjustments must come from some combination of positions in the risky and risk-free assets with a net zero cost. Call this combination of positions the hedge position. To have zero cost, the hedge position must have equal size long and short positions. Writing z as the size of long and short positions, we get:

$$a(x-r)t = z[1+x(1-t)] - z[1+r(1-t)] \quad (5)$$

The left hand side is additional tax due from the tax on the full return. The right hand side represents a long position of z units of the risky asset with a return of x and a tax of t on that return and a short position of the risk-free asset with a return of r and a tax of t on that return. Solving for z , we get $z = at/(1-t)$, as above.⁷

To complete the model note that the government's situation is exactly the opposite. If it imposes a tax on the full return, it will expect to receive an additional $a(x-r)t$ from the

⁷Often the portfolio adjustments are stated as an increase in the risky asset from a to $a/(1-t)$ and a decrease in the risk-free asset from $(1-a)$ to $(1-a/(1-t))$. See Warren (1996). This is the same adjustment as in the text.

individual over what it would have received with a tax on just the risk-free return. Suppose it taxes only the risk-free return but wishes to receive the extra $a(x-r)t$. It can produce this revenue by taking the opposite position from the individual, selling and buying the assets that the individual would like to buy and sell. Therefore, government revenues are the same under the two regime, the total net investment in each asset remains fixed, and the two taxes are equivalent. Note that because of the offsetting nature of the portfolio adjustments by the individual and the government, markets clear and market prices do not change. Note also that the government is playing no special role in absorbing risk, and government spending is not affected by the choice of tax regimes.

It is important to note that the strong nature of equivalency used here means that there is no deadweight loss from the portfolio shifts. Under a tax on the full rate of return, it might nominally look as if the individual has increased his risky positions and decreased his risk-free positions relative to a tax on the risk-free return and relative to the no tax world. This is a tax induced change in behavior that might be viewed as creating deadweight loss. But the individual's after-tax position is the same as in the world with a tax only on the risk-free return, which means the individual's well being is the same under both tax regimes. There is no deadweight loss associated with portfolio shift because of the nominal tax on the risky return (other than transactions costs).

One way to think of the portfolio adjustment is that the individual really hasn't made an adjustment to his actual investments. Instead, he costlessly finances his tax obligation, leaving his actual investments in place, by buying securities from or selling securities to the government. All of the proceeds from this costless set of positions go to the government. It is as if there is a separate bank account or pocket that is only nominally owned by the individual. The government will get any return from that bank account and the individual is indifferent. The government, however, is also short that same bank account, which means the whole thing is really a nullity. Choosing a tax nominally on the full return as opposed to a tax nominally on just the riskless return is just like choosing to require individual to go through the trouble to set up this extra bank account with no other equity or efficiency results. Similarly, the government appears to receive additional revenue from the tax on the full return, but this is just the revenue the government would get from taking risky market positions and it has zero risk adjusted net present value.

The model is stated in the case of only two assets. It trivially extends to the case of multiple assets all taxed at the same rate. The individual makes a similar portfolio adjustment for each asset depending on the holdings of that asset, leaving only a tax on the risk-free rate of return.

Before considering extensions, it is worth comparing the model and conclusions here (and in Kaplow (1995)) to those found in much of the literature. The literature emphasizes

that an income tax will affect the overall riskiness of the individual's portfolio. This is consistent with the statement here that an income tax is equivalent to a tax on the riskless return because individuals may change their risk positions due to the tax on the risk-free return – the wealth elasticity of demand for the risky asset may not be zero. The apparent difference in the statements is that the comparison here is of an income tax on the full return to a tax on the riskless return while in most of the literature the comparison is to the no tax world.

Finally, it is commonly stated that both an income tax and a consumption tax impose a tax on so-called inframarginal returns.⁸ These are defined as investments in which one cannot invest more or gross up to offset the effect of the tax. In the current model there are no inframarginal returns because the government is always supplying offsetting securities. Regardless of how attractive the investment, taxpayers will be able to invest enough more to eliminate the tax on the risky return.⁹

II. Multiple Tax Rates

The question is whether or how these results extend to the case where different types of capital are taxed at different tax rates. The nature of the portfolio adjustments in the single tax case gives the intuition. In the single tax rate case, the taxpayer finances the tax on the risky return by borrowing and buying risky assets. The risk-free borrowing and the risk-free component of the risky position exactly offset, leaving only the risky return. By entering into the right amount of this hedge position, taxpayers can finance the tax on the risky return. But if the tax rates are different, the risk-free borrowing and the risk-free component of the risky position will not exactly offset. The difference, which can be viewed as a pure, riskless arbitrage profit or loss, is the cost or benefit of eliminating the tax on the risky position.

This section illustrates this argument. First, it shows that if the tax on the risk-free rate of return is the same for all assets, the tax rate on the risky return is irrelevant – the portfolio adjustments work exactly the same way regardless of the tax rate on the risky return because the risk-free components continue to perfectly offset one another. Then this section considers the more general case of differential tax rates on the full return. The cost of offsetting the risk is, as indicated above, related to the differential rates on the risk-free rate of return for the two assets. Finally, this section discusses how this result extends to different methods of taxing assets, such as through depreciation deductions or through special rates on capital gains or losses.

⁸See, for example, Warren (1996).

⁹Kaplow (1995) has further useful comments on incentive-based reasons for the inability to trade securities in certain types of businesses.

A. *Different Tax Rates on Risky and Risk-free Returns*

Suppose that the tax system had explicit and potentially different taxes on the risky and risk-free returns to investments. For example, Bradford (1995) would impose a tax on the risk-free rate of return similar to that found in Auerbach (1991) and a separate tax (at an arbitrary rate) on any gains or losses beyond the riskless return. Other proposals, such as Shuldiner (1992), explicitly separate the risky and risk-free components of some types of investments but impose the same rate of tax on each. Sandmo (1977) models a special case where there are multiple assets taxed at different rates on the risky return, and a zero tax rate on the risk-free return (which he calls “net” taxation).

It is relatively straightforward to see that in a system of this sort, the tax on risk can be eliminated through portfolio adjustments, leaving just the tax on the risk-free returns. That is, the Bradford (1995) system is not really an extension of Auerbach (1991) in the sense that the tax on risk is a nullity. It produces no efficiency or fairness gains but comes at considerable administrative cost.

To see this, suppose that the tax on the risky return is t_1 and the tax on the risk-free return is t_0 . Wealth in period 1 without any portfolio adjustment in such a world would be

$$W_t = W - a(x-r)t_1 - rt_0 \quad (6)$$

If only the risk-free return were taxed, the wealth in period 1 would be

$$W_{rf} = W - rt_0 \quad (7)$$

The difference between (6) and (7) is $a(x-r)t_1$. This can be financed costlessly. To set this, set the long and short positions equal in size and solve:

$$a(x-r)t_1 = z[(1+x) - (x-r)t_1 - rt_0] - z[(1+r) - rt_0] \quad (8)$$

The first term in square brackets on the right hand side is the investment in the risky asset less taxes (separated into the tax on the risky return and the tax on the risk-free return). The second term is the short sale of the risk-free asset. Solving for z , we get, $z = at_1/(1-t_1)$ exactly as in the base case. The intuition should be relatively clear. The portfolio adjustments are costless because the risk-free borrowing and the risk-free component of the risky return face the same tax rate and, therefore, exactly offset.

Note that because the tax on the risk can be costlessly eliminated through portfolio adjustments and because the government has exactly the opposite exposure, the government

can supply the necessary securities through portfolio adjustments of its own that also offset the risk. Once again, therefore, the model holds in a general equilibrium context.

B. *Explicit, Non-neutral Capital Income Taxation*

The more interesting case is where different types of capital are taxed at different rates. This describes current law. Current law imposes widely different rates on different types of capital through imperfect measurement of capital income, the corporate tax, and explicit subsidies to favored industries.¹⁰ The extent to which capital is taxed at different rates has varied over the history of the income tax but at no point in time has there been a uniform tax on capital.

As noted above, we should not expect the portfolio adjustments to be costless in this case because the risk-free components will not match. This section first shows this to be the case – taxpayers cannot costlessly finance the tax on the risky position. Another possibility (the one that I initially guessed to be true) is that differential rates are equivalent to a weighted average tax on the risk-free return. This also turns out to be false, as shown here. Finally, this section illustrates the actual cost of eliminating the tax on risk.

Suppose there are only two assets, a risky and risk-free asset and that a tax of t_1 applies to the return on the risky asset and a tax of t_0 applies to the return from the risk-free asset. With such a tax and no portfolio adjustments, wealth in period 1 would be

$$W_t = W - ax_{t_1} - (1-a)rt_0 \quad (9)$$

We can rewrite this as

$$W_t = W - rt_0 - a(x_{t_1} - rt_0) \quad (10)$$

The first two terms on the right hand side are the familiar tax on the risk-free rate of return. The last term is the tax on the risky return. It cannot be costlessly financed. To see this, let z be the size of the hedge position. If the hedge position is to have a zero cost, it must have the same size long and short positions, so that:

$$a(x_{t_1} - rt_0) = z[1+x(1-t_1)] - z[1+r(1-t_0)] \quad (11)$$

Solving for z , we get

$$z = a(x_{t_1} - rt_0)/[x(1-t_1) - r(1-t_0)] \quad (12)$$

¹⁰See Jorgenson and Yun (2001), Treasury (2000).

The hedge, however, does not work because it depends on x , which is a random variable. If the hedge position must be fixed in advance (in period 0), the taxpayer must pick a value of x to determine z . But x will not always equal the chosen value, which means that the hedge will not finance the tax for every realization of x . For example, if z is based on the expected value of x , the hedge position finances the tax only if the actual realization is equal to the expected value. If x turns out differently than expected, the position will either over or underfund the tax. To see this, let x^e be the expected value of x and x' be the realized value. The hedge position produces:

$$\left(\frac{at_1(x^e - r)}{x^e(1-t_1) - r(1-t_0)} \right) [x'(1-t_1) - r(1-t_0)] \quad (13)$$

The first term is the size of the hedge, or z . The second term is the return. This expression equals $a(xt_1 - rt_0)$ if and only if either (i) x' equals x^e , (ii) r equals 0, or, trivially, (iii) t_1 equals t_0 . If x^e equals x' , the hedge worked as expected, so the tax on risk is eliminated. If t_0 equals t_1 , there is no differential taxation. Finally, if r is zero, there is no differential taxation of the risk-free portion of the return (regardless of the nominal tax rate, there is no tax because there is no return). As established above, in this case, taxpayers can eliminate the tax on risky returns when risk-free returns are taxed at the same rate. Thus, the key is the differential rate on the risk-free portion. Note that the expected tax is eliminated and any residual can just as well be a payment from the government as to the government. Nevertheless, the taxpayer is not in the same position as he would be with a tax only on the risk-free return.¹¹

How the hedge performs depends on the relative tax rates. If the tax rate on the risky position is less than the tax rate on the risk-free position, the hedge “over performs” in the sense that when the risky position does badly, the hedge provides more money than necessary to pay the tax and when the risky position does well, the hedge provides insufficient funds. Therefore, a taxpayer facing a tax on the full return but with the hedge is in a less risky position than a taxpayer facing a tax only on the risk-free return even though the taxpayer has the same expected return. This reflects the lower tax on the risky position. The lower tax on the risky position produces a better return (adjusted for risk). The opposite is true if the tax rate on the risky position is greater than the rate on the risk-free position. The hedge underperforms and the risky position will look relatively less attractive.

¹¹Note that if the taxpayer’s positions can be continuously adjusted, the taxpayer might be able to fully offset the tax liability. The ability to make such adjustments, however, is a very different sort of portfolio adjustment than that usually considered in the literature.

An alternative possibility is that a differential tax on the full return to capital is the same as a weighted average (by type of capital and tax rate) tax on the risk-free rate of return. That is, we can rewrite (9) as

$$W_t = W - [art_1 + (1-a)rt_0] - a(x-r)t_1 \quad (14)$$

The second term on the right hand side is a weighted average tax on the risk-free return. The last term is the tax on risk. Once again, this tax can be costlessly financed only on average. The argument is identical to the argument made immediately above. The hedge under or over-performs in the same circumstances.

The reason the risky position cannot be costlessly financed is that the risk-free borrowing and the risk-free component of the risky return do not offset. Nevertheless, the tax on the risky return can be offset at a cost or subsidy equal to the difference in these amounts. From (9),

$$W_t = W - rt_0 - a(xt_1 - rt_0) \quad (15)$$

The first two terms are the usual tax on the riskless rate of return on the entire portfolio. The taxpayer can finance the last term by purchasing $a t_1 / (1 - t_1)$ of the risky asset and selling $a t_0 / (1 - t_0)$ of the risk-free asset. The taxpayer will have a net receipt from entering into this hedge if t_0 is higher than t_1 and will have a net payment if the opposite is true. Let the cost (either positive or negative) of this position be H , so that:

$$aH = a[t_0 / (1 - t_0) - t_1 / (1 - t_1)] \quad (16)$$

In period 0, the taxpayer incurs aH to buy the hedge. In period 1, the hedge position will yield:

$$a(xt_1 - rt_0) + aH \quad (17)$$

The first term covers the tax on the risky portion of the taxpayer's portfolio, as required, and the second term is a recovery of the original cash price of the hedge. Therefore, entering into aH finances the tax on the risky return at the cost of aH .

The cost of aH is a time value cost because aH is incurred in period 0 and returned in period 1. Because it is returned in all circumstances in period 1, the risk-free rate of return is the proper discount rate and the cost of aH is merely aHr . Therefore, we can think of a differential income tax as a tax on the risk-free return to all capital (at the rate nominally imposed on the risk-free asset), plus a penalty or subsidy equal to the risk-free return on the

investment in the risky asset weighted by the difference in the tax rates (using a tax-exclusive rate). Writing this as a formula, we get:

$$W_i = W_{rf} - ar[t_0/(1-t_0) - t_1/(1-t_1)] \quad (18)$$

The opportunity sets for the two taxes are the same. Any position available for a tax implemented through the left hand side of (18) (a nominal tax on the entire return) can be achieved in a tax implemented through the right hand side (a tax on the risk-free return plus a fixed subsidy or penalty for investing in the risky asset).

Note that the cost (or subsidy) for investing in the risky asset as compared to a uniform tax on capital is not at all based on risky outcomes. Another dollar invested in the higher-taxed asset imposes a fixed cost based solely on the differences in tax rates and the risk-free rate of return. Just as in the Domar/Musgrave case with uniform taxation, we should not think of the income tax, even with differential rates, as imposing a tax on the risky return. Instead, we can think of differential rates as creating a fixed incentive to invest in the lower taxed asset, unrelated to risk.¹² That is, an income tax with differential tax rates should not be thought of as an ex post tax. Instead, it can be seen as a purely ex ante tax that is not at all dependent on outcomes.

The key difference between this case and the Domar/Musgrave case is that there is a profit (or cost) to the arbitrage when tax rates are different. The arbitrage looks risky on a stand alone basis, but the risk in the arbitrage exactly offsets the risk of tax payments on the taxpayer's risky asset, leaving a pure, riskless profit (or cost). The arbitrage profits or costs get very large as the tax rate on the riskless return goes up because the size of the profit or cost is based on the difference in the tax exclusive rates, which go up with $1/(1-t)$. Unlike a normal risk-free arbitrage, which is essentially unlimited, however, the arbitrage here is strictly limited in size.

There is some question of whether the tax system would allow this arbitrage. Current law contains a number of rules that prevent taxpayers from making tax profits off of riskless arbitrages. While the tax system modeled here is very abstract and one can imagine anti-arbitrage rules that disallow the arbitrage, current law would probably allow it in many cases. The reason is that the hedge looks risky – taken alone it looks like a real investment. It is only risk-free because it offsets the tax liability on the remainder of the taxpayer's risky positions. Moreover, the hedge would be integrated with the taxpayers actual investments –

¹² Another way to think about it is as the government charging a price to absorb risk. Suppose, for example, that the tax rate on the risky asset is less than the tax rate on the risk-free asset. The costless hedge in this case over performs. Instead, the taxpayer can choose to receive cash and have a perfect hedge. The cash is the (opportunity) price of the risk reduction from the costless hedge. The question is whether this a market price for risk.

the taxpayer would appear merely to have a bigger (or smaller) risky investment partially financed by debt.

The government portfolio adjustments necessary for equivalence are more problematic than in the case with a single tax rate. The reason is that taxpayer responses will vary based on the utility function of the individual taxpayer – they will respond individually to the benefit of receiving Hr when investing another dollar in the lower-taxed asset. To make offsetting adjustments, the government would have to predict the extent to which the differential tax leads to investment in the lower-taxed asset. Although this may be predictable on average, it requires more information than in the base case.¹³

The model used only two assets, but the extension to multiple assets is almost trivial.¹⁴ Without working through the algebra, suppose that there were three assets (two risky and a risk-free) and three tax rates. The investor puts a in risky asset 1, b in risky asset 2, and $1-a-b$ in the risk-free asset. We can make essentially identical portfolio adjustments with the net cost being the same (for the given tax rates) in each case.

C. *Other Tax Patterns*

The model so far is intended to be of securities that produce a clearly identifiable return taxed at a single rate (which might be different than the rate imposed on other securities). While this might sometimes resemble actual assets, differential tax rates are often imposed through much more complex mechanisms, such as accelerated depreciation, tax credits, or differential tax rates applied to different pieces of the return on a single security. The adjustments (or whether adjustments can be made) will depend on the details of implementation. Because of the wide variety of possible patterns, it is not possible to consider every possibility. Instead, I will consider two relatively important cases: (i) the case where differential tax rates are imposed through depreciation or tax credits, and (ii) the case where differential tax rates are imposed on different pieces of the return from a single security, such as where some of the return is treated as a capital gain.

1. *Depreciation and Credits*

There are a number of different ways to model risk in depreciable assets and exactly how it enters may affect how and whether the tax on risk can be offset. Consider the

¹³Note, however, that if the government can make the proper adjustments the problem of differential responses to the tax system does not undermine the claim of equivalence. The opportunity sets facing taxpayers are the same under the two regimes, and, therefore, taxpayer behavior should be identical.

¹⁴Note that the two asset case did not impose any restrictions (beyond concavity) on the individual's utility function, unlike those found in Tobin (1958), which might have made the extension to multiple assets more complicated.

following simple case. Assume that the government sets a depreciation schedule in advance and the schedule may or may not equal economic depreciation. The extent to which it differs from economic depreciation determines the extent to which the tax on the asset is greater than or less than the nominal rate.¹⁵ Assume also that the cash flows from the asset are risky but because the depreciation schedule is set in advance, the tax benefit from depreciation is not. In this case, any difference between expected cash flows and actual flows will be taxed at the nominal rate. For example, suppose immediate (partial) expensing is allowed and the cash flows from the investment will be either \$50 or \$150. Any difference between the expected value of \$100 and the actual value will face a tax at the nominal rate.

We can think of the difference between non-economic depreciation and economic depreciation as a present value amount for a dollar investment. Let this amount equal K . Because K is certain, it would be computed using the risk-free rate of return. If we think of K as a present value deduction, its value is Kt . (If alternatively we modeled a credit, its value would be K .) Following the notation from above, a taxpayer's wealth if he invests a in the risky asset would be:

$$W_t = W - (1-a)rt - axt + aKt \quad (19)$$

If instead of a nominal income tax, the government imposed a tax on the risk-free rate of return, the taxpayer's wealth would be

$$W_{rf} = W - rt \quad (20)$$

Once again, we can decompose the tax on the full return into a tax on the risk-free component and a tax on the risky component, and exactly as above, the tax on the risky component can be financed for free. Thus, the only difference between the two cases is the aKt , the penalty or subsidy from the differential taxation. There is no effective tax burden on the risky component of the return.

Note also that we can relate this formula to the formula for explicitly different rates to determine a marginal tax rate that is equivalent to any given pattern of non-economic depreciation. That is, we can derive a marginal effective tax rate that takes into account portfolio adjustments induced by nominal taxes on risk. To do this, suppose that the nominal rate on all capital is t , and we know K . The explicit rate t_{ETR} that would give the same incentives as that pattern of depreciation is defined by the equation:

¹⁵ Assume that if the depreciation schedule is slower than economic depreciation, the taxpayer cannot sell and repurchase the asset in each period to claim a loss, thereby reducing tax depreciation to economic depreciation. If the taxpayer can sell the asset and alter the pattern of tax depreciation, the difference between tax and economic depreciation is risky and the model would have to be altered. For discussion of risky depreciation and strategic selling, see Strnad (1999).

$$Kt = r[t_{ETR}/(1-t_{ETR}) - t/(1-t)] \quad (21)$$

Note that this looks quite different from the usual marginal effective tax rate formulas with accelerated depreciation that do not account for risk.¹⁶

2. *Different Rates on Components of the Return*

An alternative case is where different components of the return from an asset are taxed at different rates. (I will call this a component tax.) For example, regular cash flows might be taxed as ordinary income while appreciation or depreciation might be taxed as capital gain or loss. To model this, suppose a risky asset has two types of (independent) random returns, x and y , taxed at t_1 and t_2 respectively. In addition, suppose that the risk-free asset is taxed at t_0 . An individual's wealth in such a regime would be:

$$W_t = W - (1-a)rt_0 - axt_1 - ayt_2 \quad (22)$$

Taxpayers in this world can eliminate any expected tax on the risky return. There is, however, no way to completely eliminate the tax on the risk. That is, there is no way to restate this tax as a tax purely on the risk-free rate of return (plus or minus a fixed cost to account for the differential tax rates).

To see that taxpayers can eliminate the expected tax on risk, decompose the total return into risky and risk-free components. Rewriting (22), we get

$$W_t = W - rt_0 - a(xt_1 + yt_2 - rt_0) \quad (23)$$

The first two terms are the usual tax on the risk free return. To finance the last set of terms, the taxpayer can enter into a costless hedge position of size z such that:

$$a(xt_1 + yt_2 - rt_0) = z[1 + x(1-t_1) + y(1-t_2)] - z[1+r(1-t_0)] \quad (24)$$

As before, the hedge does not work in all cases if it must be set in advance because it depends on x and y . For example, if z is set using the expected values of x and y , the hedge will be effective if and only if the realized values of x and y are equal to the expected values (or trivially, the tax rates are the same).¹⁷

¹⁶See, for example, Auerbach (1982) for a sample of typical effective tax rate formulas.

¹⁷Note that unlike the case with a single tax on each security, the hedge does not work merely because r equals zero. The reason why is that the single hedge cannot eliminate both the x and y risk.

Taxpayers cannot completely eliminate the tax on risk, even at a cost, through portfolio adjustments. To see this, note that to eliminate the tax on risk, there must be a fixed hedge position such that the risky return on the hedge position equals the risky tax obligation in all states of nature. That is, there must exist w such that w is not dependent on x or y and

$$w[x(1-t_1) + y(1-t_2)] = xt_1 + yt_2 \quad (25)$$

Inspection shows that no w satisfies this unless x is perfectly correlated with y (so that there really is only one component) or the tax rates are the same. There is no position in the risky asset such that the after-tax return correlates perfectly with tax payments. Therefore, a differential component tax cannot be restated as a tax on the risk-free rate of return. It similarly cannot be stated as an ex ante tax – it will inevitably depend on outcomes.

Note that this conclusion depends on an inability of taxpayers to buy and sell each type of risk, x and y , independently. The reason taxpayers cannot eliminate the risk is because they need different size hedges to offset the different x and y risks but are constrained to a single hedge of size z . If they can separately buy and sell either x or y , the case reverts to the prior differential rate cases discussed above.

It is interesting to compare these conclusions with the conclusions of Bulow and Summers (1984). They argue that much of the risk in depreciable assets comes from the risk of price changes rather than the risk of cash flow changes. They then argue that the tax system only taxes cash flows, not price changes. In such a world, the government taxes all the receipts or flows from depreciable assets but absorbs none of the risk. Depreciable assets subject to tax, therefore, will have a lower return but the same risk, which means taxes have a negative effect on risk bearing, even if depreciation perfectly matches economic depreciation.

In the model here, we can view the two components of return, x and y , as cash flows and price changes, respectively. If price risk is not taxed, t_2 would equal zero, and the model will then apply. The taxpayer can enter into the hedge of size z and costlessly eliminate any expected taxation of risk, leaving only an equal chance of a subsidy or tax on actual returns. The system does not look like Bulow and Summers world, where government taxes cash flows but leaves risk.

The reason why the conclusion here seems to differ from Bulow and Summers is that in their model, *all* of the return is taxed through the tax on cash flows while *none* of the risk is exposed to tax. That is, all of the compensation for bearing price risk comes in the form of additional cash flows rather than as future capital gains. The model above implicitly assumed that each component of the return properly priced for risk. If prices reflect the present value

of expected future cash flows and all cash flows are taxed, it seems hard to support the Bulow/Summers model.¹⁸

We can generalize the Bulow and Summers result. The result is dependent on the price of the differently taxed components not individually reflecting their risk. In their example, the cash flows are fixed and are fully taxed even though some portion of the cash flows compensate the investor for price risk. This allows the government to tax all the return on the asset but absorb none of the risk. We can imagine the other cases. Suppose the government only taxed the price risk and not the cash flows. Then the government would be absorbing risk but not taking any of the return. It would be like free insurance. Between these extremes, one can imagine almost allocation of risk, return, and tax to different components of an asset. One again, it is not clear the extent to which any of these patterns are likely – a better starting assumption would be that each component of return on an asset is properly priced.

D. *Summary*

We can summarize the results so far as follows.

1. A uniform income tax on the entire return to capital is equivalent to a tax on just the risk-free return to capital.
2. An explicit, differential tax on the entire return to capital is equivalent to a uniform tax on the risk-free return to all capital plus a fixed payment for the purchase of the lower-taxed asset. The payment for an additional dollar invested in the lower-taxed asset is equal to the risk-free rate of return multiplied by the difference in the tax-exclusive tax rates.
3. Combining 1 and 2, a differential tax on the entire return to capital is a uniform tax on the entire return plus a fixed payment.
4. A differential tax imposed through noneconomic depreciation can be translated into an explicit differential tax (and then conclusions 2 and 3 apply).

¹⁸See Strnad (1999) for additional comments on the Bulow and Summers model. Strnad argues that Bulow and Summers have it almost exactly backwards. The reason is that if the asset performs unexpectedly badly, taxpayers can sell the asset, claim a loss and force the government to share in the downside risk. If the asset performs unexpectedly well, the taxpayer can hold the asset and prevent the government from participating in the upside. Thus, taxpayers holding risky assets are better off rather than worse off because of the way price risk is taxed.

5. A differential component tax when the components are fairly priced (but cannot be separately traded) cannot be restated as an ex ante tax. Taxpayers, however, can eliminate any expected tax on the risky returns.
6. A component tax when the components are not fairly priced individually may impose a tax or subsidy to risk, depending on the pricing of the components and the tax rates that apply to each component.

III. Implications and Comments

This section discusses implications of the models and provides additional comments. It highlights three implications. The first is whether or to what extent implications from single rate Domar/Musgrave models carry over. The second implication is the size of the deadweight losses from differential taxation. The third is on accuracy and measurement.

A. Nature of Income Taxes

The results concerning taxation and risk are often used to argue that there is little difference between an income tax and a consumption tax. The reason is that a tax on the risk-free rate of return is likely to be very small because the risk-free rate of return itself is small.¹⁹ In addition, the result has been used to devise tax systems that are equivalent to an income tax but that might be easier to administer.²⁰ The question whether these results carry over to differential income taxes.

To make the comparison, one has to specify exactly which taxes are being compared. Suppose we are comparing an income tax with different rates applying to different types of capital with a cash flow consumption tax (say at the business level) with similar rate differentials. We know a differential income tax is equivalent to a tax on the risk-free rate of return plus a subsidy or penalty for investing in particular assets. The consumption tax, even with differential rates, imposes a zero tax on new capital.²¹ Differential rates make no difference in a consumption tax because the gross-up used to eliminate the tax on capital comes from the deduction when an asset is purchased and does not depend on an arbitrage between differentially taxed assets. Therefore, the difference between a differential income

¹⁹See, for example, Gentry and Hubbard (1997), Bankman and Griffith (1992).

²⁰See, e.g., Auerbach (1991).

²¹Note that differential consumption tax rates may have effects on transition and for inframarginal returns.

tax and a differential consumption tax is the tax on the risk-free rate of return plus or minus the subsidies or penalties under the differential income tax.²²

If there is a differential component tax, the argument might be more complex. One would have to model differential component consumption taxes – a beast I have not yet encountered. Although it would take further study, intuition suggests that taxpayers would be able to gross up their portfolios to avoid the tax because, again, the gross up does not depend on an arbitrage. The comparison is made more complex because of the difficulty in characterizing an income tax of this sort.

Another implication of the Domar/Musgrave type models is that income taxes have no ex post fairness benefits (at least with respect to the capital tax portion). That is, it might have been thought that one of the advantages of an income tax is that it taxes more heavily those who are lucky than those who are unlucky.²³ If an income tax is just a tax on the risk-free rate of return, luck has no effect on tax liabilities. This remains true with differential rates (again, depending on how they are imposed) – differential income taxes can be thought of as purely ex ante taxes.

Finally, although harder to find in the literature but commonplace in discussions, the Domar/Musgrave analysis means failure to index for inflation becomes relative much more important than we might have thought because inflation is a relatively larger component of the tax on capital. For example, suppose we thought the risky portion of return were taxed. If inflation is low, say two percent, the risk-free rate of return is two percent (real), and the risky return is ten percent, failure to index for inflation may not look that important because inflation is a relatively small component of the total return. If, however, the risky return is not taxed, a tax on inflation becomes relatively much larger. In this example, where the risk-free return and inflation are at the same rate, failure to index for inflation doubles the tax rate on real returns! It is likely that at recently seen levels of inflation, the tax rate on the real, risk-free return would exceed 100%. This conclusion remains with differential rates, subject to the same caveats.

²²Some commentators have broken down a switch from a nonuniform income tax to a uniform consumption tax as a two step process in which rate differentials are first eliminated and then the tax on the risk-free return is eliminated. See Gentry and Hubbard (1998). The logic is similar to that in the text.

²³See, for example, Graetz (1979).

B. *The Costs of Differential Rates*

It is common for the literature to examine effective tax rates on capital, showing that there are major differentials.²⁴ One of the major themes of the 1986 Tax Reform Act was to reduce these differentials. The literature rarely incorporates risk into the calculations even though it invariably applies to risky assets.

Statements about tax rate differentials must be interpreted with caution. As shown above, a differential between a risky and risk-free asset can be recast as a fixed subsidy for the lower-taxed asset and a uniform rate on the risk-free rate of return. The size of the subsidy will determine the effect of the differential tax rate. The size of the subsidy, however, varies with the tax exclusive rate, which means that absolute and relative differences in tax-inclusive rates can be misleading. In particular, differences between tax-inclusive rates become relatively (much) more important as rates go up. At low rates, seemingly large difference may matter very little.

For example, suppose the risk-free rate of return is 3%, the tax on the risk-free asset is 40%, and the tax on the risky asset is 0%. An additional dollar invested in the risky asset produces only a two cent subsidy.²⁵ If the tax rate on the risk-free asset were 62.5%, however, the tax on the risky asset would have to be 50% to produce the same subsidy. The absolute and relative differences do not look the same but the behavioral effect is the same. Small differences in high rates can have very large effects, while large differences in low rates may have very little effect.

Most commentators do not translate tax rate differentials into deadweight losses. Instead, they present the differentials as if this alone were enough to draw conclusions. Mere tax rate differentials, of course, are not enough, but deadweight loss calculations are very difficult. We do not know the elasticities of substitution between asset types. There is a considerable literature measuring the elasticity of overall investment to changes in tax rates, but this data is not sufficient to determine how investment patterns are distorted because of differential tax rates.²⁶ Those who have attempted estimates have use only very rough estimates.²⁷

²⁴For recent examples, see Mackie (2002), Jorgenson and Yun (2001), and Treasury (2000), Gravelle, (1994).

²⁵From (18), $0.03 (0.4/(1-0.4)) = 0.02$.

²⁶See Hassett and Hubbard (2002), for a survey.

²⁷See, for example Fullerton and Henderson (1989), and Jorgenson and Yun (2001).

To the extent we have data on elasticities of substitution between asset types, it is not clear that the analysis here changes the calculation of deadweight loss. If measurement of the elasticities is based on actual changes in asset use when tax rates change, this data could be used to calculate deadweight loss directly – we simply need changes in capital stocks for changes in price. The end result would have to be consistent with the deadweight loss one would see from a uniform tax plus an ex ante subsidy, and, therefore, the analysis here can act as a check. For example, the analysis here suggests that elasticities should change as tax-inclusive rates go up because small differences in high tax-inclusive rates produce large subsidies. In addition, the analysis suggests that we might expect low deadweight losses even for what looks like relatively large differentials as long as overall rates remain low.

C. *Costs and benefits of accuracy and measurement*

One of the most important problems facing designers of tax systems is the extent to which resources should be spent measuring taxable attributes.²⁸ Just as with the deadweight loss determinations, the analysis here suggests that if overall rates remain low, accuracy in measuring capital income may be unimportant.²⁹

For example, it is very difficult to measure depreciable lives of many investments. There is considerable controversy over the legal standard to be applied and even more controversy over its application to particular expenditures. For example, there have been significant problems determining the treatment of expenses that produce only intangible benefits, such as expenses to reorganize a workforce or to improve reputation. It is hard to capture all these expenses in an accounting system and even harder to determine the useful life of the expenses. Nevertheless, many of these expenditures clearly produce long lived benefits that should, in theory, be capitalized. The Treasury Department reports that for large corporate taxpayers, 25% of exam resources are now spent on capitalization issues and has announced that it intends to issue regulations simplifying the area.

If overall rates are low, the analysis here suggests that measurement often may not be worthwhile. For example, if the risk-free rate is 40% and the risk-free return is 3%, allowing an immediate deduction for hard to capture capital expenditures is equivalent to taxing these accurately and granting a two cent per dollar subsidy for making these expenditures. It is very likely that measurement costs significantly exceed this subsidy. Measurement costs act as a kind of penalty – if one is going to incur an expenditure that brings with it high

²⁸See Kaplow (1996,1998) for discussion of this issue.

²⁹There may be reasons relating to tax shelters and arbitrage transactions for accurate measurement. Discussion of these issues is beyond the scope of this paper.

measurement costs, these costs will be added to the overall costs of the investment and, therefore, reduce the return. The deadweight loss from the subsidy may be far less than the deadweight loss from the measurement costs. Greater accuracy may actually increase distortions.

D. *Further comments*

This section makes three additional comments. First, there is some question about the ability of taxpayers and the government to make the portfolio adjustments suggested here and in the large Domar/Musgrave literature. As formulated here and in Kaplow (1995), the problem is one of incomplete markets. Taxpayers may not be able to buy or sell additional units of every asset and government may not be able to do the opposite. If this is true, the adjustments necessary to offset the tax on risk will not be possible. For example, imagine that the taxpayer's risky asset is a farm. If all the land nearby is already being used, the taxpayer may not be able to buy additional units of the farm to finance the tax on his return to risk.

We might be tempted to think of this as upward sloping supply problem. Additional units of the farm or other asset are available only at a greater cost. Under the models considered here, the problem is not increased demand for any asset. Instead, the problem is that taxpayers and the government cannot enter into offsetting transactions in the asset. If markets were complete, the government could then sell derivative positions in the farm, and the taxpayer could purchase the positions.³⁰

Markets are not complete (and transactions costs are not zero), so taxpayers and the government will not be able to fully offset the tax on risk in all cases. Moreover, even where markets are liquid, the government does not appear to adjust its portfolio to offset the effects of taxation on risk. But it not clear the extent to which this is a serious deviation from the model. Markets are now able to provide a huge variety of risk positions and they are becoming more complete every day. Moreover, taxpayers can adjust to risk in a variety of subtle ways. For example, natural hedges can be removed and the structure of debt or other financing can be changed in subtle ways. Governments can also adjust the riskiness of its overall revenues with actually trading in securities markets. They can provide subsidies or penalties based on outcomes, they can provide guarantees, and they can adjust its ownership in actual assets. We wouldn't have to see the actual separate accounts or government trading imagined in the models to believe that taxpayers and the government adjust in many ways to the tax on risk.

³⁰In models where the government does not make offsetting portfolio adjustments, taxpayer portfolio adjustments may lead to increased demand for risky assets and upward sloping supply might indeed be a problem.

Second, the model does not consider loss limitations. Because loss limitations limit government participation in downside risk, they might affect the conclusions here. Some of the Domar/Musgrave type models incorporate loss limitations³¹ while others have argued that they are quantitatively unimportant.³²

One way to incorporate loss limitations into the models in this paper is to note that one of the purposes of loss limitations is to reduce the tax rate on losses because of the likely low rate on gains stemming from the taxpayer's option to defer gains. That is, without loss limitations and even with the same nominal rate, losses would be deducted at a higher effective tax rate than gains would be taxed because losses would be realized early and gains deferred. Loss limitations reduce this disparity in effective tax rates.³³ To the extent this is the case, the combination of loss limitations and deferral of gains acts merely as a reduction in the overall rate of tax on an asset, leaving approximately equal sharing of gains and losses with the government. The effect would obviously be crude – loss limitations are a highly imperfect method of achieving this, but nevertheless, it may not be that far from correct.

Third and finally, there is a reasonably large literature on the tax adjusted capital asset pricing model. This literature does not incorporate Domar/Musgrave type results.³⁴ Under a uniform tax and with government portfolio adjustments, one would suspect that there should be no change in the CAPM with and without tax because the CAPM is a measure of the return to risk and risk is not affected by an income tax. With differential rates, risk premiums should still not change because the benefit of differential rates is unrelated to the risk of an investment. More work would be required, however, to incorporate the Domar/Musgrave results explicitly into the CAPM.

IV. Conclusion

The paper extends the analysis of the effect of income taxation on risk to the case where different types of assets are taxed at different tax rates. This extension is important because differential income taxation is endemic. The key result is that taxpayers can eliminate the tax on risk even where there are differential tax rates with the cost or subsidy of doing so based on the differences in the tax rate imposed on the risk-free rate of return. The

³¹See Atkinson and Stiglitz (1980).

³²See Gordon (1985).

³³See Johnson (1993) for this argument.

³⁴See Poterba (2002) for a survey.

nature of the portfolio adjustments, their cost, and the implications, however, depend on the exact pattern of taxation for a particular asset.

References

- Atkinson, Anthony, and Stiglitz, Joseph, *Lectures on Public Economics*, New York: McGraw-Hill, 1980
- Auerbach, Alan, *Tax Neutrality and the Social Discount Rate*, *Journal of Public Economics* 17 (1982): 355-372.
- Auerbach, Alan, *Retrospective Capital Gains Taxation*, *American Economic Review* 81 (March 1991): 167-78.
- Bankman, Joseph and Fried, Barbara, *Winners and Loses in the Shift to a Consumption Tax*, *Georgetown Law Journal* 86 (January, 1998): 539-568.
- Bankman, Joseph, and Griffith, Thomas, *Is the Debate Between and Income Tax and a Consumption Tax a Debate About Risk? Does it Matter?*, *Tax Law Review* 47 (Winter, 1992): 377-406
- Bradford, David, *Fixing Realization Accounting: Symmetry, Consistency, and Correctness in the Taxation of Financial Instruments*, *Tax Law Review* 50 (1995): 731-786.
- Bulow, J.I., & Summers, L.H., *The Taxation of Risky Assets*, *Journal of Political Economy* 92 (1984): 20-39.
- Domar, E.D., & Musgrave, R.A., *Proportional Income Taxation and Risk-Taking*, *Quarterly Journal Economics* 58 (1944): 388-422.
- Feldstein, Martin, *The Effects of Taxation on Risk Taking*, *Journal of Political Economy* 77 (September/October 1969): 755-64.
- Fullerton, Don and Henderson, Yolanda, *The Marginal Excess Burden of Different Capital Tax Instruments*, *Review of Economics and Statistics* 79 (August 1989): 435-442.
- Gentry, William, and Hubbard, R. Glenn, *Distributional Implications of Introducing a Broad-Based Consumption Tax*, in *Tax Policy and the Economy* 11 (James Poterba ed., 1997) 1-__.
- Gentry, William and R. Glenn Hubbard, *Fundamental Tax Reform and Corporate Financial Policy*, in J.M. Poterba, ed., *Tax Policy and the Economy*, volume 12, Cambridge: MIT Press, 1998.

- Gordon, R.H., *Taxation of Corporate Capital Income: Tax Revenues versus Tax Distortions*, Quarterly Journal of Economics 100 (1985):1-27.
- Gravelle, J.G., *The Economic Effects of Taxing Capital Income* (MIT press, Cambridge, MA) (1994).
- Graetz, Michael, *Implementing a Progressive Consumption Tax*, Harvard Law Review, 92 (1979): 1575-1661.
- Hassett, Kevin, and Hubbard, R. Glenn, *Tax Policy and Business Investment*, in Handbook of Public Economics 3 (Alan Auerbach and Martin Feldstein, eds., 2002): 1293-1343.
- Johnson, Calvin, Commentary, *Deferring Tax Losses with an Expanded Section 1211*, Tax Law Review 48 (1993): 719-__.
- Jorgenson, Dale, and Yun, Kun-Young, *Investment Volume 3: Lifting the Burden*, (2001)
- Kaplow, L., *Taxation and Risk-Taking: A General Equilibrium Perspective*, National Tax Journal 47 (1994): 789-798.
- Kaplow, Louis, *How Tax Complexity and Enforcement Affect the Equity and Efficiency of the Income Tax*, National Tax Journal 49 (1996): 135-__.
- Kaplow, Louis, *Accuracy, Complexity, and the Income Tax*, Journal of Law, Economics, and Organization 14 (1998): 61-83.
- Mackie, James, *Unfinished Business of the 1986 Tax reform Act: An Effective Tax Rate Analysis of Current issues in the Taxation of Capital Income*, National Tax Journal 55 (June 2002): 293-337
- Mossin, J., *Taxation and Risk-Taking: An Expected Utility Approach*, Economica 35 (1968): 74-82.
- Poterba, James, *Taxation, Risk Taking and Household Portfolio Behavior*, in Handbook of Public Economics 3 (Alan Auerbach and Martin Feldstein, eds., 2002): 1110-1171
- Sandmo, Agnar, *Portfolio theory, Asset Demand, and Taxation: Comparative Statics with Many Sssets*, Review of Economic Studies 44 (1977): 369-379.

Shuldiner, Reed, *A General Approach to the Taxation of Financial Instruments*, Texas Law Review 71 (1992): 243-350.

Stiglitz, J.E., *The Effects of Income, Wealth, and Capital Gains Taxation on Risk-Taking*, Quarterly Journal of Economics 83 (1969): 262-283.

Strnad, Jeff, *Tax Depreciation and Risk*, SMU Law Review 52 (Spring 1999): 547-654 (1999).

Tobin, James, *Liquidity Preference as Behavior towards Risk*, Review of Economic Studies 25 (February, 1958): 65-86.

U.S. Department of the Treasury, *Report to the Congress on Depreciation Recovery Period and Methods*, Washington, D.C.: U.S. Government Printing Office July 2000.

Warren, Alvin, Jr., *Would a Consumption Tax Be Fairer Than an Income Tax?*, Yale Law Journal 89 (1980): 1081-__.

Warren, Alvin, Jr., *How Much Capital Income Taxed Under and Income Tax Is Exempt Under a Cash Flow Tax?*, Tax Law Review 52 (Fall 1996): 1-16.