Valuing Identity:

The Simple Economics of Affirmative Action Policies*

Roland G. Fryer, Jr. Glenn C. Loury

Revised: February 2007

Abstract

Affirmative action policies are practiced around the world. This paper explores the welfare economics of such policies. A model is proposed where heterogeneous agents, distinguished by skill level and social identity, compete for access to scarce positions. The problem of designing an efficient policy to raise the success rate in this competition of a disadvantaged identity group is considered. We show that: (i) when agent identity is fully visible and contractible (sightedness), efficient policy grants preferred access to positions, but offers no direct assistance for acquiring skills; and, (ii) when identity is not contractible (blindness), efficient policy lowers productivity requirements across the board, randomly rations access to positions and, under plausible conditions, entails a universal skills subsidy.

*We are grateful to Pierre-Andre Chiappori, Bernard Salanie and two anonymous referees for helpful comments and suggestions. We also thank seminar participants at numerous universities. Fryer is at the Department of Economics, Harvard University, and NBER, 1875 Cambridge Street, Cambridge MA 02138 (e-mail: rfryer@fas.harvard.edu); Loury is at the Department of Economics, Brown University, 64 Waterman Street, Providence RI 02912 (e-mail:Glenn_Loury@brown.edu). Loury gratefully acknowledges financial support from the Andrew W. Mellon Foundation and the Carnegie Corporation. Fryer thanks the DuBois Institute for African and African-American Research and the Institute of Quantitative Social Science for generous support.
“This is the next and the more profound stage of the battle for civil rights. We seek not just freedom but opportunity. We seek not just legal equity but human ability, not just equality as a right and a theory but equality as a fact and equality as a result.” President Lyndon B. Johnson, Howard University, 1965

1 Introduction

When productive or developmental opportunities must be rationed in a population, the social identities of those selected can be a matter of great importance. By “identity” we refer – varying with the application – to an agent’s race, sex, age, nationality, religion, ethnicity, or caste. When choosing which students to admit, employees to hire, candidates to slate, or firms to patronize, government and business actors alike often confront an intense public demand for some intervention that will engender more diversity in the ranks of the chosen. As a consequence, regulations intended to achieve this end – policies going under the rubric of “affirmative action,” or “positive discrimination,” or (less neutrally) “reverse discrimination” – have been promulgated in many societies throughout the world. This paper examines the welfare economics of such diversity-promoting public regulation.

Consider a few examples from around the globe. In nations with sharp sectarian divisions – Lebanon, Indonesia, Pakistan, Iraq – political stability can hinge on maintaining ethnic balance in the military ranks, or on distributing coveted political offices so that no single group has disproportionate influence. In the US, selective colleges and universities often feel obliged to alter their admissions standards to enhance the racial diversity of their student bodies. In Europe, some political parties have mandated that female candidates be adequately represented on their electoral lists. In post-Apartheid South Africa, to ensure that wealth is distributed across a wider spectrum of society a policy of “Broad Based Black Economic Empowerment” has been enacted, setting minimum numerical standards of black representation that companies are obliged to meet. In Malaysia, in the wake of widespread ethnic rioting that erupted in 1969, a “New Economic Policy” was instituted, creating quotas and preferences for ethnic Malays in public contracting, employment, and education. In India, so-called “scheduled castes and tribes” enjoy preferred access to university
seats and government jobs by constitutional mandate, though amidst fierce controversy.\(^1\) When there is no confusion, we will refer to all the diversity enhancing efforts as affirmative action.

### 1.1 Some Economic Questions

There has been much heated debate about the *fairness* of affirmative action. Yet, these policies – and the controversy they inevitably inspire – can be found virtually everywhere. For this reason – while the fairness issues are a real concern – we focus on how greater racial or ethnic diversity in highly prized positions can be *efficiently* achieved.\(^2\)

Typically, affirmative action policies aim to enhance the presence of a “disadvantaged” social group in some competitive pursuit. They entail the preferential valuation of social identity based on a presumption that, on the average, those being preferred cannot compete on an equal basis because of a pre-existing (if not innate) social handicap. But the stubborn realities of unequal development create some unavoidable economic problems: In the short-run, at least, enhanced access for a genuinely disadvantaged group to much sought-after productive opportunities cannot be achieved without lowering standards, distorting human capital investment decisions, or both.\(^3\)

So, given that the supply of opportunity is limited, exogenous between-group differences necessarily make diversity a costly commodity. The relevant economic problem then becomes understanding how these costs should be conceptualized, and how they can be minimized.

Our analysis of the welfare economics of affirmative action policies is motivated by two particular thematic questions:

1. Where in the economic life cycle should preferential treatment be most emphasized – before or after productivities have been essentially determined?

2. How do public policies that valorize a non-productive trait – i.e., identity – affect private

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\(^1\)Many other examples could be given, from countries such as Phillipines, Nigeria and Sri Lanka. For a comprehensive review of the use of these policies in global perspective, see Thomas Sowell (2004). On maintaining ethnic diversity in military selection, see Klitgaard(1986). On racial preferences at US colleges and universities, see Bowen and Bok (1998). On caste and ethnic preferences in India, see Galanter (1992) and Deshpande (2006).

\(^2\)For a discussion of the social justice issues raised by affirmative action and other racially egalitarian policies, see Loury (2002), Chp. 4.

\(^3\)See Fryer and Loury (2005a) for a detailed discussion of some usually overlooked, yet unavoidable, trade-offs associated with affirmative action policies.
incentives to become more productive?

1.2 Our Approach

To explore these questions, we develop a simple economic model of preferential identity valuation. A population of agents belonging to distinct social groups invest in human capital and then compete for assignments that give them an opportunity to use their skills. One group is disadvantaged, and policies to enhance opportunity for the agents in that group are considered. Designing an efficient policy of this kind is posed as an elementary problem of optimal taxation. Our analysis is novel, relative to the existing literature on affirmative action, in our focus on the second-best efficiency question, and in the attention we give to the visibility and the timing dimensions of these policies.4

As just noted, the timing issue has to do with finding an ideal point in the developmental process to introduce a preference. We distinguish in the model between the ex ante and the ex post stages of production. Given that a productivity gap already exists, an ex post preference offers a competitive edge to less productive agents in the disadvantaged group. By contrast, an ex ante preference promotes the competitive success of the disadvantaged by fostering their prior acquisition of skills. That is, ex ante policies operate on the development margin; while ex post policies operate on the assignment margin. This distinction is important for the design of preferential policies because agents do not have the same incentive to develop skills when they anticipate being preferred in subsequent assignments.

The visibility dimension of a policy concerns an informational constraint one often encounters with affirmative action, reflected in the distinction we draw between sighted and blind policy environments. Under sightedness, assignment standards and development subsidies can be tailored to group membership at the individual level. Sighted policies are overtly discriminatory, in that otherwise similar agents from different groups are treated differently. Blind policies, in contrast, are tacitly discriminatory. They have their impact by placing a premium on some non-identity traits that are known to be more prevalent in the preferred population. Though they are facially

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4 Earlier papers on the economics of affirmative action policies include Welch (1976), Lundberg and Startz (1983), Coate and Loury (1993), Moro and Norman (2003), and Fryer and Loury (2005b). For a review of the evidence on the effectiveness of these policies, see Holzer and Neumark (2000). For a broad policy discussion, see Fryer and Loury (2005a).
neutral in their treatment of groups, blind preferential policies have been intentionally chosen to have group-disparate effects.

Combining these distinctions of visibility and of timing generates a $2 \times 2$ conceptual matrix that captures the main contours of affirmative action as it is practiced in the real world: Job reservations, contract set-asides, distinct admissions standards, race-normed ability tests – all exemplify sighted-ex post preferences. Instances of sighted-ex ante preferences include minority scholarship funds, group-targeted skills development programs, and costly outreach and recruitment efforts that encourage an underrepresented group to prepare for future opportunities.

On the other hand, automatic admission for the top 10% of a state’s high school classes; waiving a mandate that college applicants submit test scores; selecting among applicants partly by lot; or introducing non-identity factors that are unrelated to performance into the evaluation process – are all examples of blind-ex post preferences. And, since there must be some group disparity in the distribution of endowments (otherwise, no policy promoting group equality would be needed), a blind-ex ante preference can always be put in place by subsidizing for everyone those skill-enhancing actions from which agents in a preferred group can derive the most benefit.5

A hybrid policy environment is also conceivable – one that is sighted/ex ante but blind/ex post. The view – popular in some circles in the US – that using race in admissions to institutions of higher education is always wrong (blindness ex post), but resources can legitimately be expended to narrow a racial performance gap in secondary schools (sightedness ex ante), illustrates this hybrid approach.6

1.3 Some Answers

Our analysis sheds considerable light on the questions raised above. Take the issue of timing: If policy-making over the economic life cycle is uncoordinated, early-stage proponents of diversity

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5Chan and Eyster (2003) have shown that lotteries can be used to pursue affirmative action goals when racial identity is not contractible. Fryer and Loury with Yuret (in press) generalize this result and, using data on US college admissions, go on to estimate the efficiency losses from adopting blind rather than sighted preferential policies at the ex post stage. Fryer and Loury (2005b) study blind handicapping in the context of winner-take-all tournaments.

6For arguments consistent with this hybrid view, see the work of Thernstrom and Thernstrom (1997) and (2003). One might imagine that there must be two such hybrid scenarios. Yet, we show below that ex ante visibility is irrelevant to efficient policy design when the regulator is sighted, ex post.
may overcompensate for a group’s social disadvantage by failing to take due account of subsequent efforts. Indeed, our model makes it clear that an explicit ex ante preference is always redundant in a sighted environment when the ideal ex post preference is in place.

Likewise, consider the issue of incentives: Economic intuition suggests that, under an efficient policy, the marginal social cost of selecting another disadvantaged agent should be the same, whether that is done by lowering productivity requirements at the assignment stage, or by raising investments in productivity at the development stage. Under sightedness, as just noted, private incentives naturally comport with this rule when ex post policies are efficient. However, under blindness, private and social returns on ex ante investments need not coincide. In fact, we can use our model to show that private investment incentives are socially inadequate in a blind policy environment whenever members of the targeted group are relatively more likely to be found on the ex ante development margin than on the ex post assignment margin.\(^7\)

The next section of this paper introduces a simple model of production, with investment in skills and competition for scarce positions. The following section brings affirmative action policy into the model, to formally represent public interventions that expand opportunity for a disadvantaged group. We then define, and explicitly characterize, efficient policies under alternative visibility regimes. In the conclusion, we discuss some of the implications and the limitations of our theoretical results.

\section{The Model}

Our model abstracts from many institutional details in order to focus on the pure economic logic of the problem. We imagine a world where “agents” are assigned to “slots” to produce “widgets.” These agents form a continuum of unit measure, and belong to one of two identity groups, \(i \in \{A, B\}\). Each is endowed with a cost of effort, \(c \geq 0\), independently drawn from a probability distribution that depends on identity. The fraction of agents in group \(i\) is \(\lambda_i \in (0, 1)\), with \(\lambda_a + \lambda_b = 1\).\(^8\)

\(^7\)Under this condition, then, requiring identity preferences to be both efficient and blind obligates one to promote the general development of skills in the overall population! Moreover, when the stated condition fails, private investment incentives are socially excessive, and efficiency requires that ex ante skills acquisition be universally \textit{taxed}!

\(^8\)In subscripts we designate identity with the lower case \((i = a, b)\).
Economic activity takes place in two stages. At the ex ante stage agents, distinguished by their identity and effort cost \((i, c)\), choose whether or not to exert effort. An example of this type of effort could be entering a training program, where the training cost can be seen as an inverse measure of the agent’s endowed capacities. At the ex post stage these same agents, now distinguished by identity and productivity \((i, v)\), compete for access to slots, and production takes place. Here a slot could be a contract, a job, or some other scarce and remunerative professional opportunity; and productivity can be seen as a measure of the agent’s acquired ability to make use of that opportunity.

The two stages of production are linked, in that productivity in a slot, \(v \geq 0\), is taken to be a noisy function of prior effort at the individual level. Let \(e \in \{0, 1\}\) be an agent’s effort choice; and, let \(\alpha \in [0, 1]\) be the probability that an agent is assigned to a slot, and thus has an opportunity to produce. An inelastic supply of slots, with measure \(\theta < 1\), must be rationed at the ex post stage.\(^9\) In this context, to expand “opportunity” for a disadvantaged group means to ensure that more of its members are assigned to slots. The technology of production is such that one agent combines in fixed factor proportions with one slot to produce one widget, the market value of which equals the productivity of that agent. Figure 1 illustrates the sequence of actions that we envision.

[FIGURE 1 GOES ABOUT HERE]

### 2.1 Notation and Assumptions

The primitives of this model are the distributions of agents’ costs and productivities. Let \(G_i(c)\) be the probability that a group \(i\) agent is endowed with an effort cost that is less than or equal to \(c\). Then,

\[
G(c) \equiv \lambda_a G_a(c) + \lambda_b G_b(c)
\]

is the effort cost distribution for the entire population. Let \(g_i(c)\) and \(g(c)\) be the respective, continuous density functions. These distributions are assumed to have a common, connected support. The inverse functions, \(G_i^{-1}(z)\) and \(G^{-1}(z)\), \(z \in [0, 1]\), give the effort cost of an agent at the \(z^{th}\) quantile of the respective populations. By the Law of Large Numbers and our continuum of agents

\(^9\)An upward-sloping supply curve for slots would complicate but not alter our results, as long as affirmative action aims merely to get more disadvantaged agents into slots, and not to redistribute rents.
assumption, $G_i(c)$ is also the fraction of agents in group $i$ with effort cost less than or equal to $c$. We assume that group $B$ is *disadvantaged* relative to group $A$, in the following sense:

**Assumption 1:** $\frac{g_a(c)}{g_b(c)}$ is a strictly increasing function of $c$.

Monotonicity of this likelihood ratio implies that, for $c$ interior to the cost support:

1. $G_a(c) > G_b(c)$
2. $\frac{G_a(c)}{G_a(c)} > \frac{g_a(c)}{g_b(c)} > \frac{1-G_a(c)}{1-G_b(c)}$; and,
3. $\frac{G_a(c)}{G_a(c)}$ and $\frac{1-G_a(c)}{1-G_b(c)}$ are both strictly decreasing functions of $c$.

Given effort, $e \in \{0,1\}$, let $F_e(v)$ be the probability that an agent’s ex post productivity is no greater than $v$. These distributions are also assumed to have a common, connected support. Their continuous density functions are denoted by $f_e(v)$. For a mass of agents with the common effort level, $e$, $F_e(v)$ is the fraction of that mass with productivity less than or equal to $v$. Exerting effort raises stochastic productivity in the following sense:

**Assumption 2:** $\frac{f_1(v)}{f_0(v)}$ is a strictly increasing function of $v$.

As before, monotonicity implies, for $v$ interior to the productivity support:

1. $F_1(v) < F_0(v)$;
2. $\frac{F_1(v)}{F_0(v)} < \frac{f_1(v)}{f_0(v)} < \frac{1-F_1(v)}{1-F_0(v)}$; and,
3. $\frac{F_1(v)}{F_0(v)}$ and $\frac{1-F_1(v)}{1-F_0(v)}$ are both strictly increasing functions of $v$.

Consider now the distribution of productivity in a population where the fraction of agents who exerted effort is $\pi$. For $0 \leq \pi \leq 1$ and $v \geq 0$, define:

$$F(\pi, v) \equiv \pi F_1(v) + (1-\pi) F_0(v).$$

Let $f(\pi, v)$ be the density and define the inverse, $F^{-1}(\pi, z)$, $z \in [0,1]$, as the productivity level at the $z^{th}$ quantile of this ex post distribution.
Finally, let \( \pi_i \) be the fraction of group \( i \) agents who exert effort. Then \( F(\pi_i, v) \) is the ex post distribution of productivity in that group, and

\[
\lambda_a F(\pi_a, v) + \lambda_b F(\pi_b, v) = F(\lambda_a \pi_a + \lambda_b \pi_b, v) \equiv F(\pi, v)
\]

is the corresponding productivity distribution for the population as a whole. Given Assumption 2, it is easily verified that:

\[
\pi_a > \pi_b \text{ implies } \frac{f(\pi_a, v)}{f(\pi_b, v)} \text{ is a strictly increasing function of } v.
\]

### 2.2 Resource Allocation

If there were no policy intervention of any kind, then more productive agents would out-compete the less productive for assignment to slots. So, a disadvantaged group would be underrepresented (holding a fraction of the slots that is less than their fraction in the population), since their effort costs are higher (Assumption 1), and effort is productive (Assumption 2). We refer to this null policy regime, where no attempt is made to assist the disadvantaged, as *laissez-faire*.

In order to characterize the equilibrium allocation of resource under *laissez-faire*, we suppose that agents act to maximize their net expected payoffs; that slots are privately held; and that access to them is traded on a competitive market.\(^{10}\) At the ex post stage, given their prior choices and the resolution of uncertainty at the individual level, variably productive agents \((i, v)\) enter this market to compete for slots which are inelasically supplied. Buyers of slots then produce and earn the value of their widgets. Let \( p^m \) be the market clearing price. At the ex ante stage, while anticipating the slots price, variably endowed agents \((i, c)\) make effort decisions. Let \( \pi^m \) be the fraction choosing \( e = 1 \).

In market equilibrium, obviously, the measure \( \theta \) of the highest value producers gain access to slots at a price equal to the productivity level at the \((1 - \theta)^{th}\) quantile of the ex post distribution. Thus:

\[
p^m = F^{-1}(\pi^m, 1 - \theta).
\]  

(1)

Therefore, the surplus to an ex post agent of type \((i, v)\) is \( \max\{v - p^m; 0\} \). So, if an ex ante agent of type \((i, c)\) exerts effort, and thereby shifts his productivity distribution from \( F_0(v) \) to \( F_1(v) \), his

\(^{10}\)Or, if slots are publicly held, then we suppose that access is sold at competitive auction.
expected gross return from doing so is:

$$\int_{p^m}^{\infty} (v - p^m)(f_1(v) - f_0(v))dv = \int_{p^m}^{\infty} \Delta F(v)dv.$$ 

where $\Delta F(v) \equiv F_0(v) - F_1(v) \geq 0$. It follows that:

$$\pi^m = G(\int_{p^m}^{\infty} \Delta F(v)dv).$$  \hspace{1cm} (2)

Equilibrium under *laissez-faire* is therefore defined by the unique values of $\pi^m$ and $p^m$ for which (1) and (2) simultaneously hold. Figure 2 depicts the determination of equilibrium in this simple model.

[FIGURE 2 GOES ABOUT HERE]

This equilibrium is socially efficient, in that no other feasible allocation of resources generates a greater surplus of widget values over effort costs. More formally, let an *allocation* be a pair of functions, $e(i, c) \in \{0, 1\}$ and $\alpha(i, v) \in [0, 1]$, specifying the effort and assignment probability of each type of agent, at each stage of production. An allocation is *feasible* if it assigns a mass of agents to slots that does not exceed $\theta$:

$$\sum_{i=a,b} \lambda_i \int_0^{\infty} \alpha(i, v)f(\pi_i, v)dv \leq \theta, \hspace{1cm} \text{(3)}$$

where $\pi_i = \int_0^{\infty} e(i, c)g_i(c)dc$ is the group $i$ effort rate. And, it is *efficient* if it maximizes net social surplus,

$$\sum_{i=a,b} \lambda_i \{ \int_0^{\infty} v\alpha(i, v)f(\pi_i, v)dv - \int_0^{\infty} ce(i, c)g_i(c)dc \},$$  \hspace{1cm} (4)

among all feasible allocations.

To solve this maximization problem defining efficiency, let the fraction of agents exerting effort in some allocation be $\pi \in [0, 1]$. Clearly, efficiency requires that only the top $\theta$ productivity quantiles be assigned to slots, and only the bottom $\pi$ cost quantiles exert effort. So, the aggregate of widget values in this allocation is, at most:

$$Q(\pi, \theta) \equiv \int_{1-\theta}^{1} F^{-1}(\pi, z)dz;$$

and, the aggregate of effort costs is, at least:

$$C(\pi) \equiv \int_0^{\pi} G^{-1}(z)dz.$$
We conclude that this allocation can be efficient only if its effort rate equals $\pi^*$, where:

$$\pi^* \equiv \arg \max_{0 \leq \pi \leq 1} \{Q(\pi, \theta) - C(\pi)\}.$$  \hspace{1cm} (5)

With an additional, mild assumption, the first-order condition from (5) is necessary and sufficient for efficiency.\footnote{The objective in (5) is strictly concave, since the RHS of (6) (marginal benefit) decreases and the LHS (marginal cost) increases with $\pi$. Furthermore, it is easily verified that an interior solution for (5) exists if and only if the cost and productivity distributions satisfy:

(i) $\inf \{c \mid G(c) > 0\} < \int_{F_0^{-1}(1-\theta)}^{\infty} \Delta F(v) dv$

and

(ii) $\sup \{c \mid G(c) < 1\} > \int_{F_1^{-1}(1-\theta)}^{\infty} \Delta F(v) dv$.}

Taking the required derivatives, we have:

$$G^{-1}(\pi^*) = \int_{1-\theta}^{1} \frac{\partial F^{-1}(\pi^*, z)}{\partial \pi} dz = \int_{F^{-1}(\pi^*, 1-\theta)}^{\infty} \Delta F(v) dv.$$  \hspace{1cm} (6)

where the second equality above follows from the Implicit Function Theorem, after changing the variable of integration. Comparing (6) with (1) and (2), one sees immediately that these are equivalent conditions. So, $\pi^m = \pi^*$, and the equilibrium allocation is efficient.

The economic logic here, though straightforward, is worth highlighting explicitly: Equilibrium in the ex post slots market generates consumer’s surplus for higher-productivity buyers. The increment to these rents that is anticipated from exerting effort provides the incentive for agents to incur costs at the ex ante stage. In competitive equilibrium, this private incentive equals the social return from greater effort, ensuring efficiency. In symbols:

$$\int_{p^m}^{\infty} (v - p^m)(f_1(v) - f_0(v)) dv = \frac{\partial Q}{\partial \pi}(\pi^m, \theta).$$  \hspace{1cm} (7)

The LHS of (7) is the private return from effort at equilibrium prices, and the RHS is the social marginal return, given the equilibrium effort rate. So, the market clearing price $p^m$ is the Lagrange multiplier on the constraint (3) in the maximization of (4) that defines efficiency. As we shall see, a similar logic applies in the case of sighted, but not blind, affirmative action.
As noted, because they have uniformly higher costs, group B agents exert effort at a lower rate than group A. So, A’s hold slots at a higher rate than do B’s. Indeed, let $\pi^*_i$ be group i’s effort rate in the efficient (and the equilibrium) allocation, and define $\rho^*_i$ to be the rate at which group i agents hold slots. Then, for $\pi^*$ given in (5):

$$
\pi^*_i = G_i\left( \int_{F^{-1}(\pi^*,1-\theta)}^{\infty} \Delta F(v)dv \right) \text{ and } \rho^*_i = 1 - F(\pi^*_i,F^{-1}(\pi^*,1-\theta))
$$

$i = a,b$. So, it follows from Assumptions 1 and 2 that:

$$
\pi^*_a > \pi^* > \pi^*_b \text{ and } \rho^*_a > \theta > \rho^*_b.
$$

Summarizing, we have:

**Proposition 1** The equilibrium allocation of resources under laissez faire is socially efficient. Given our assumptions, the disadvantaged group (B) exerts productivity-enhancing effort, and gains access to productive opportunities, at a lower rate than does the advantaged group (A) in this equilibrium.

These results are no great revelation, but deriving them explicitly allows us to introduce concepts that will prove useful in the analysis to follow.

### 3 Valuing Identity

The unequal outcome in (8) is what affirmative action policies are designed to “correct.” Since this laissez-faire allocation is efficient, any departure from it in the interest of identity diversity must reduce social surplus.

Suppose that a central authority (the regulator) wants to raise the fraction of group B agents who acquire slots to some target level, $\rho_b \in (\rho^*_b,\theta]$. We envision the following sequence of events: The regulator commits to a policy. Agents receive their endowments and make their effort choices. Productivities are realized. A slots market equilibrium is reached. Then, widgets are produced.

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12 Throughout this paper, the affirmative action target is taken to be exogenous. We study the consequences of alternative ways to achieve it. Moreover, we rule out any target more aggressive than one that aims for population parity which, in any case, is the most ambitious goal that an identity-blind policy could ever realize.
and their values earned. Our problem is to characterize a \textit{constrained-efficient} affirmative action policy – one that is anticipated to achieve the regulator’s representation target, $\rho_b > \rho^*_b$, in a surplus-maximizing manner.

We must first be more specific about what we mean by an “affirmative action policy.” Assume that the regulator observes the agents’ actions, but not their costs or productivities. His interventions are therefore limited to mandating transfers – to or from the agents – that depend on whether effort is exerted, and on whether a slot is held. Assume further that the regulator can impose an upper bound on the price at which slots trade in the ex post market, and that he requires random rationing among willing buyers whenever there is excess demand. Then an \textit{affirmative action policy} is simply a list of the (action×identity)-contingent transfers, and the upper bound on the price of slots, to which the regulator is committed. This policy is blind if its transfers do not depend on identity.

Social scientists used to thinking about affirmative action in terms of “quotas” may doubt that this tax/subsidy formulation corresponds closely enough to real world practice. Yet, if the goal is to understand economic issues that bear on incentives and efficiency, we would argue that the fit is really quite good. Conventional duality considerations imply that any system of quantity constraints (quotas) can be associated with an implicit tax/subsidy scheme (derived from the shadow prices on those constraints) such that, if agents trade freely at tax-inclusive prices then, in the ensuing market equilibrium, an equivalent outcome obtains (up to a lump-sum redistribution of rents). For this reason, we believe that the core economic implications of policies like the ones we described in the Introduction – blind and sighted, ex ante and ex post – are adequately captured in this abstract model of regulation.

\subsection{3.1 Sighted Environments}

Let us now consider how such policies could affect resource allocation in a sighted environment. A crucial implicit assumption that we are making in this analysis is that the regulator has no interest in redistribution as such (either between himself and the agents, or among the agents). We take it that he wants merely to raise the slots access rate of $B’$s, to some level $\rho_b > \rho^*_b$, in an efficient manner. Notice also that the binary choices which the agents make here (exert effort or not; buy a slot or not) can depend only on transfer-inclusive payoff differences. No generality is lost, therefore,
by supposing that the transfers to agents who do not exert effort and do not acquire a slot are zero.

Similarly, with the supply of slots being perfectly inelastic, a universal transfer to all slot holders
would simply be capitalized into the slots price. All that can matter for ex post resource allocation
is the difference between identity groups in the magnitude of transfers to slot holders. Again ignoring
rent shifting (now between the regulator and slot owners), no generality is lost by supposing that
the transfer to group A slot holders is zero, as well.

Therefore, in a sighted environment an affirmative action policy is fully determined by three
numbers – $(\sigma_a, \sigma_b, \tau)$ – denoting, respectively, the regulator’s transfers to the A’s and B’s who exert
effort, and to the B’s who hold slots. Ex ante policy under sightedness operates on the development
margin, enhancing opportunity for B’s either by subsidizing their effort, or by taxing the effort of
A’s. And, ex post policy operates on the assignment margin, by enabling lower productivity B’s
to outbid higher productivity A’s in the competition for slots.

Let some sighted policy $(\sigma_a, \sigma_b, \tau)$ be given. Then, in the ensuing market equilibrium, agents
from group $i$ exert effort at some rate, $\pi_i$, and slots trade at some price, $p$, such that:

$$1 - \theta = \lambda_a F(\pi_a, p) + \lambda_b F(\pi_b, p - \tau),$$

and such that:

$$\pi_a = G_a(\sigma_a + \int_p^\infty \Delta F(v)dv)$$

$$\pi_b = G_b(\sigma_b + \int_p^{p-\tau} \Delta F(v)dv).$$

Equation (9) is the requirement that the slots market clears. And, equations (10a) and (10b) state
that the agents make rational effort choices. Moreover, if B’s hold slots at the rate $\rho_b \in (\rho_b^*, \theta]$, and
if the slots market clears, then A’s must hold slots at the rate $\rho_a \in [\theta, \rho_a^*]$, where:

$$\lambda_a \rho_a + \lambda_b \rho_b = \theta.$$  (11)

So, if the regulator’s target, $\rho_b$, is to be met in this equilibrium, then the slots price, $p$, and the ex
post transfer to group B slot holders, $\tau$, have to be such that:

$$p = F^{-1}(\pi_a, 1 - \rho_a) \quad \text{and} \quad p - \tau = F^{-1}(\pi_b, 1 - \rho_b)$$  (12)

where, from (11), $\rho_a \equiv [\theta - \lambda_b \rho_b]/\lambda_a$. 

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It follows that a sighted regulator never needs to impose a price ceiling. His problem may be recast as choosing effort rates for the two groups, subject to his representation target, and then finding a pure transfer policy \((\sigma_a, \sigma_b, \tau)\) that causes those rates to arise in the ensuing equilibrium. To see this, notice first that in a sighted environment social surplus is fixed once a representation target, \(\rho_i\), and the effort rates, \(\pi_a\) and \(\pi_b\), are given. The top \(\rho_i\) quantiles in each group compete successfully for slots, which determines the aggregate value of widges produced; and, the bottom \(\pi_i\) quantiles exert effort, which determines total effort costs. Thus, let an arbitrary pair of effort rates, \((\pi_a, \pi_b)\), be given. Then in light of (10a), (10b), (11), and (12), the unique sighted policy \((\sigma_a, \sigma_b, \tau)\) which induces those effort rates to arise in the ensuing market equilibrium is as follows:

\[
\begin{align*}
\sigma_i &= G_i^{-1}(\pi_i) - \int_{F_i^{-1}(\pi_i, 1 - \rho_i)}^{\infty} \Delta F(v) dv, \quad i = a, b; \text{ and} \\
\tau &= F_i^{-1}(\pi_a, 1 - \rho_a) - F_i^{-1}(\pi_b, 1 - \rho_b)
\end{align*}
\]

Thus, let \((\sigma^*_a, \sigma^*_b, \tau^*)\) denote the efficient sighted affirmative action policy. The argument to this point has established the following useful characterization:

**Lemma 1** In a sighted environment, let \((\pi^*_a, \pi^*_b)\) be the group-specific effort rates that maximize net social surplus, subject to the constraints that group \(i\) agents hold slots at the rate \(\rho_i, i = a, b\). Then \((\sigma^*_a, \sigma^*_b, \tau^*)\), the efficient affirmative action policy, is defined by (13) with \(\pi_i = \pi^*_i, i = a, b\).

### 3.2 Blind Environments

By contrast, consider the impact of affirmative action policy in a blind environment. Now regulatory transfers cannot vary with identity. Since any uniform transfer to slot holders gets capitalized into the slots price, the only way a blind regulator can promote \(B\)'s access to slots at the ex post stage is to hold the price down below its market clearing level, and randomly ration slots among agents willing to buy at this artificially low price. This brings in agents just below the assignment margin, and excludes an equal number of infra-marginal agents. By Assumption 2, the former are more likely to be disadvantaged than are the latter. So, a blind ex post preference lowers productivity standards by moving from “select the very best” to “select those who are good enough.” The more modest is the “good enough” threshold, the higher will be the rate at which \(B\)'s get assigned to slots.
Blind policy at the ex ante stage works by imposing an across-the-board tax or subsidy on effort. This either narrows or widens, respectively, the set of agents who exert effort. How such a policy affects slot assignment rates turns on the relative numbers in the two groups of agents who are at or near the development margin. If many more B’s than A’s have costs just a bit too high to warrant exerting effort under laissez faire, then an effort subsidy will cause the equilibrium rate at which B’s hold slots to rise. Conversely, if B’s are comparatively scarce on the development margin, then a tax suppresses the effort of A’s by more than that of B’s and, in this way, causes the rate at which B’s hold slots to rise in equilibrium.

We conclude that a blind affirmative action policy is fully determined in this model by the two numbers — (σ, pc) — which denote, respectively, the regulator’s transfer to all agents who exert effort, and his binding upper limit on the price of slots. Let πi denote the group i effort rate; let π be the overall rate; and let p be a binding [1 – F(π, pc) > θ] price ceiling. With random rationing, the probability, αc, that a willing buyer [v ≥ pc] is assigned to a slot is:

\[ αc ≡ \frac{θ}{1 - F(π, pc)} < 1. \]  

(14)

Now, let some blind policy (σ, pc) be given. Since agents choose effort rationally, and since under blindness agents from both groups anticipate the same returns, the equilibrium induced by this policy must have effort rates satisfying:

\[ π = G(σ + αc \int_{pc}^{∞} ΔF(v)dv) \]  

and

\[ πi = Gi(G^{-1}(π)), \quad i = a, b. \]  

(16)

Moreover, B’s are holding slots at rate ρb = αc · [1 – F(πb, pc)] in this equilibrium. Therefore, using (14) and (16), the blind regulator’s representation constraint can be written as:

\[ 0 = ρb[1 – F(π, pc)] – θ · [1 – F(Gb(G^{-1}(π)), pc)] \equiv R(π, pc; ρb). \]  

(17)

Under our assumptions R(π, pc; ρb) strictly increases with pc.\(^{13}\) So, with π fixed, for every ρb ∈ (ρb, θ] there is a unique price ceiling pc which allows the blind regulator to achieve his representation target. And, the more aggressive is the target, the lower is this price ceiling.

\(^{13}\)To prove monotonicity, notice that:

\[ \frac{d}{dp}LHS = θf(πb, pc) – ρbf(π, pc) ≤ 0 \quad \text{as} \quad \frac{f(πb, pc)}{f(π, pc)} ≤ \frac{ρb}{θ} = \frac{1 – F(πb, pc)}{1 – F(π, pc)}. \]
Finally, notice that under blindness social surplus is given, once an aggregate effort rate \( (\pi) \) and a price ceiling \( (p^c) \) are specified: The population productivity distribution and aggregate effort costs are functions of \( \pi \); and agents with \( v \geq p^c \) have the probability \( \alpha^c = \frac{\theta}{1 - F(\pi, p^c)} \) of being assigned a slot, so the aggregate of widget values is also given. It follows that a blind regulator’s problem can be recast as choosing \( \pi \) and \( p^c \) to maximize social surplus, subject to the constraint (17). Given his choice of \( (\pi, p^c) \), (14) and (15) can then be used to find the unique ex ante transfer, \( \sigma \), which induces the chosen effort rate to arise in equilibrium. Indeed:

\[
\sigma = G^{-1}(\pi) - \left[ \frac{\theta}{1 - F(\pi, p^c)} \right] \cdot \int_{p^c}^{\infty} \Delta F(v) dv
\]

We have arrived at another very useful characterization of ideal policy:

**Lemma 2** In a blind environment, let \( (\tilde{\pi}, \tilde{p}^c) \) be the population effort rate and slots price ceiling that maximize social surplus subject to the constraint (17). Then the efficient affirmative action policy is \( (\tilde{\sigma}, \tilde{p}^c) \), for \( \tilde{\sigma} \) given by (18) with \( \pi = \tilde{\pi} \) and \( p^c = \tilde{p}^c \).

This, then, is our model of regulation, from which the results asserted in the Introduction immediately follow.

To summarize: a sighted regulator in effect chooses the group-specific effort rates, \( \pi_i \), to maximize surplus subject to realizing in equilibrium the group-specific representation targets, \( \rho_i \). The efficient sighted affirmative action policy \( (\sigma^s_a, \sigma^s_b, \tau^s) \) can then be determined via (13). On the other hand, a blind regulator in effect chooses an aggregate effort rate, \( \pi \), and a price ceiling, \( p^c \), to maximize surplus subject to his representation constraint (17). The efficient blind affirmative action policy \( (\tilde{\sigma}, \tilde{p}^c) \) is then defined by (18).

## 4 The Simple Economics of Affirmative Action Policies

We can now harvest the fruit of our labor by characterizing efficient preferential policies under alternative visibility regimes. Using (17). But, by Assumption 1 and (16), \( \pi_b < \pi \) in any blind equilibrium. Assumption 2 then implies

\[
\frac{f(\pi_b, p^c)}{f(\pi, p^c)} > \frac{1 - F(\pi_b, p^c)}{1 - F(\pi, p^c)}
\]

which establishes the result.
4.1 Sighted Environments

In a sighted policy environment — given the target rates at which the two groups are to hold slots in equilibrium, \((\rho_a, \rho_b)\), and in light of (4) — social surplus may be expressed, as a function of \(\pi_a\) and \(\pi_b\), as follows:

\[
\text{sighted surplus} = \sum_{i=a,b} \lambda_i \{Q(\pi_i, \rho_i) - C_i(\pi_i)\},
\]

where \(Q(\pi_i, \rho_i) \equiv \int_{1-\rho_i}^1 F^{-1}(\pi_i, z)dz\) is the sum of widget values and \(C_i(\pi_i) \equiv \int_0^{\pi_i} G_i^{-1}(z)dz\) is the per-agent cost of effort within group \(i\), when the group’s effort rate is \(\pi_i\).

Inspection of (19) makes it clear that a sighted regulator — one who can condition his policy explicitly on identity at both stages of production — really faces separate resource allocation problems for the two groups, each akin to the problem that defines efficiency under laissez faire. Instead of a single ex post limit on capacity \((\theta)\), there is now a limit for each group \((\rho_i)\), subject to which the benefits and costs of ex ante effort \((\pi_i)\) must be traded-off. Hence, first-order conditions like (6) — but with \(G_i^{-1}\) replacing \(G^{-1}\) and \(\rho_i\) replacing \(\theta\) — can be employed to deduce the efficient policy: Within each group \(i\), the marginal cost should equal the marginal benefit of ex ante effort, taking into account that only the top \(1-\rho_i\) productivity quantiles can be assigned to slots, ex post. Expressing this formally:

\[
G_i^{-1}(\pi^*_i) = \int_{F^{-1}(\pi^*_i, 1-\rho_i)}^{\infty} \Delta F(v)dv, \ i = a, b.
\]

For every representation target, \(\rho_b \in (\rho_b^*, \theta)\), (20) implies a unique pair of constrained-efficient effort rates, \((\pi^*_a, \pi^*_b)\). (See Figure 3.)

**FIGURE 3 GOES ABOUT HERE**

But then, comparing (20) with (13) and using Lemma 1, we have:

**Proposition 2** Given a representation target \(\rho_b \in (\rho_b^*, \theta)\), with \(\rho_a \equiv [\theta - \lambda_b \rho_b]/\lambda_a\), let \((\pi^*_a, \pi^*_b)\) be the unique solutions of (20). Then, the efficient sighted affirmative action policy is:

\[
\sigma^*_a = \sigma^*_b = 0 \text{ and } \tau^* = F^{-1}(\pi^*_a, 1-\rho_a) - F^{-1}(\pi^*_b, 1-\rho_b)
\]

Proposition 2 says that no explicit skills subsidy should be used to efficiently promote the access of a disadvantaged group to scarce positions when identity is fully contractible. Rather, taking
the human capital investments in the groups as given, the regulator should simply underwrite the competitive position of disadvantaged agents at the ex post stage, to the extent necessary to meet his representation target. The anticipation of this preferential treatment provides an *implicit* subsidy to skills acquisition by the disadvantaged. And, since an ex post preference for B’s raises the price of slots, it implicitly taxes the human capital acquisition of A’s by lowering their anticipated return from exerting effort. In the equilibrium, this overt policy of discrimination in favor of disadvantaged agents at the ex post stage calls forth constrained-efficient levels of effort from the agents in both groups at the ex ante stage.\(^\text{14}\)

Our finding here can be seen as a special case of a celebrated proposition in the theory of public finance due to Diamond and Mirrlees (1971): Absent any desire to redistribute firms’ profits, a system of optimal indirect taxes avoids distortions that lead to inefficient production.\(^\text{15}\) The ex ante versus ex post distinction in our set-up is equivalent to the Diamond-Mirrlees distinction between intermediate factors versus final consumption goods. Therefore, absent any concern on our regulator’s part to redistribute rents as such, and given that his ex post intervention is efficient, no additional distortions at the ex ante stage are desirable. We stress that this would continue to be the case with a more complex model of production than the one specified here – a model with skill complementarities across the agents, for example – even though the additive separability in (19) would not be preserved in the more complex setting.

### 4.2 Blind Environments

Recall from (17) that in a blind policy environment the representation constraint is: \(R(\pi, p^c; \rho_b) = 0\). Moreover, since the measure \(\theta\) of slots is assigned at random to agents with \(v \geq p^c\), social surplus under blindness is just \(\theta\) times the mean productivity of these willing buyers when the population effort rate is \(\pi\), net of the effort costs. That is:

\[
\text{blind surplus} \equiv S(\pi, p^c) = \theta \mu^+(\pi, p^c) - \int_0^\pi G^{-1}(z)dz, \quad (21)
\]

\(^{14}\)Thus, by raising the return on effort for B’s and lowering it for A’s, an efficient policy of sighted affirmative action must narrow the inter-group skills gap relative to *laissez-faire*. (That is, \(\pi_b^s > \pi_a^s\) and \(\pi_a^s < \pi_b^s\).) The phenomenon of “patronization” (see Coate and Loury, 1993), where preferential treatment undercuts incentives for the disadvantaged to acquire skills, does not occur if transfer-inclusive slots prices equate the private and social returns from effort for all agents.

\(^{15}\)We thank Bernard Salanié of Columbia University for stressing the relevance to our work of this classical result.
where

\[ \mu^+(\pi, p^c) \equiv E[v \mid v \geq p^c; \pi] = [1 - F(\pi, p^c)]^{-1} \int_{p^c}^{\infty} vf(\pi, v) dv \]

is the conditional mean productivity among willing buyers at the slots price \( p^c \).

Thus, by Lemma 2, the efficient blind policy is defined by the optimization problem:

\[ \max \{ S(\pi, p^c) \mid R(\pi, p^c) = 0 \}. \]

A necessary condition for \((\tilde{\pi}, \tilde{p}^c)\) to solve this problem is that:

\[ \frac{\partial S}{\partial \pi} \bigg| _{\tilde{\pi}, \tilde{p}^c} = \frac{\partial S}{\partial p^c} \bigg| _{\tilde{\pi}, \tilde{p}^c}. \] (22)

This says that the loss of surplus from raising the rate at which group B agents gain access to slots should be the same at the margin, whether this increased representation of B’s is accomplished by shifting the ex ante effort rate, \( \pi \), or by lowering the ex post productivity threshold, \( p^c \).

Carrying out the tedious differentiation in (22) and rearranging terms, the first-order condition which, together with (17), defines the efficient blind policy can be written as:

\[ G^{-1}(\pi) - \frac{\theta}{1 - F(\pi, p^c)} \int_{p^c}^{\infty} \Delta F(v) dv = H(\pi, p^c) \cdot K(\pi, p^c), \] (23)

for functions \( H \) and \( K \) defined as follows:

\[ H(\pi, p^c) \equiv \frac{\theta \Delta F(p^c)}{1 - F(\pi, p^c)} \cdot [\mu^+(\pi, p^c) - p^c] > 0, \]

and\(^{16}\)

\[ K(\pi, p^c) \equiv \frac{g_b - \frac{f_b}{f}}{\frac{f}{f} - \frac{g_b}{g}} \geq 0 \text{ as } \frac{g_b}{g} \lesssim \frac{f_b}{f}. \]

Here we are employing the notational short-hand:

\[ \frac{f_b}{f} = \frac{f(G_b(G^{-1}(\pi)), p^c)}{f(\pi, p^c)} \text{ and } \frac{g_b}{g} = \frac{g_b(G^{-1}(\pi))}{g(G^{-1}(\pi))}. \]

Notice that \( \frac{X}{f} \) is the relative number of B’s among agents on the ex post assignment margin (i.e., with \( v = p^c \)). And \( \frac{X}{g} \) is the relative number of B’s on the ex ante development margin (i.e., with \( c = G^{-1}(\pi) \)). So, comparing (23) with (18) in the light of Lemma 2 establishes the following result:

\(^{16}\)The inequality here relies on the fact that, under our assumptions:

\[ \frac{\rho_b}{\theta} = \frac{1 - F(\pi_b, p^c)}{1 - F(\pi, p^c)} < \frac{f_b}{f}. \]
Proposition 3  Given a representation target $\rho_b \in (\rho_b^*, \theta)$, let $(\pi, \tilde{p}^c)$ satisfy (23) and (17). Then, the efficient blind affirmative action policy entails the ex post ceiling price, $\tilde{p}^c$, and the ex ante effort-contingent transfer, $\tilde{\sigma}$, such that:

$$\tilde{\sigma} = H(\pi, \tilde{p}^c) \cdot K(\pi, \tilde{p}^c).$$

The efficient policy subsidizes human capital investment for all agents ($\tilde{\sigma} > 0$) if and only if disadvantaged agents are more prevalent on the development margin than on the assignment margin.

Under laissez-faire, because the agents anticipate a market-clearing price for slots, $p^m = F^{-1}(\pi^m, 1 - \theta)$, they make socially efficient human capital investment decisions. With an identity-blind preference at the ex post stage, however, agents see a lower than market-clearing price ($p^c < p^m$), and face the uncertainty of random rationing ($\alpha^c < 1$). When greater ex ante effort allows the representation target to be met with less rationing ex post, private investment incentives will be socially inadequate. This is because agents do not capture the benefits from raising their common probability of obtaining a slot ($\alpha^c$) which is, in effect, a public good. When the marginal investor is more likely to belong to the disadvantaged group than is the marginal producer, a greater ex ante effort rate allows the representation target to be met in such a way that $\alpha^c$ rises. So, this is the circumstance under which an efficient, blind affirmative action policy will provide a universal subsidy for the acquisition of skills.

It is interesting to reflect on the circumstances when this critical condition – that the disadvantaged are relatively more prevalent among marginal investors than marginal producers – might be expected to hold. If slots are in especially short supply ($\theta \approx 0$), the marginal producer will fall far in the right tail of the productivity distribution where (by Assumption 2) disadvantaged agents are relatively scarce. On the other hand, when the investments needed to enhance productivity in a given pursuit are especially difficult for disadvantaged agents to make ($G_b(c) << G_a(c)$), we would expect them to be relatively scarce among marginal investors. These observations suggest that blind affirmative action should employ a general subsidy for skills acquisition only if the type of opportunities in which greater diversity is being sought are not too rarefied (i.e., difficult to qualify for), but also not too plentiful.
5 Beyond the Model

Let us comment briefly on the generality of our analysis. We emphasize that the particular institutions proposed here – privately held slots and competitive ex post slots market with affirmative action policies implemented via a tax/subsidy scheme – are not essential to the results in this paper. What really matters in this analysis is the presumed asymmetry of information on costs and productivities between the regulator and the agents. Given this, and an additional assumption on the productivity distributions, the seemingly ad hoc regulatory process we have described here is no less general than a full-blown mechanism design analysis.

It can be shown that when the likelihood ratio \( \frac{f_1(v)}{f_0(v)} \) is a convex function of \( v \), then the optimal incentive compatible, two-stage mechanism – one that maximizes social surplus subject to the representation constraint – coincides in both blind and sighted environments with the outcomes described in our Propositions 2 and 3. (A proof of this claim is available from the authors upon request.) The intuition for this result is that, under the stated condition, the best allocation that can be implemented when agents’ costs and productivities are private information has a threshold property: slots are assigned with positive probability only to the most productive; and effort is exerted only by those with the lowest costs. But, given this, it follows that the net payoffs delivered by such a mechanism to agents of every type must be the same as the payoffs delivered in the equilibrium of the regulatory process examined here (up to a uniform lump-sum transfer between agents and slot owners).

One can see this by considering the mechanism design problem of implementing an efficient allocation of resources under laissez-faire in our model. Notice that any agent who exerts effort or acquires a slot can always pose as the type of agent who is at the margin of (and thus, by definition, indifferent to) doing so. Hence, by ex post incentive compatibility, a mechanism that induces ex ante effort at some rate, \( \pi \), allows any infra-marginal agent with \( v > F^{-1}(\pi, 1 - \theta) \) to achieve the ex post payoff, \( v - F^{-1}(\pi, 1 - \theta) \). And, by ex ante incentive compatibility, an infra-marginal agent with \( c < G^{-1}(\pi) \) is guaranteed the ex ante payoff of \( G^{-1}(\pi) - c \). But, these are identical to the payoffs that agents receive in our model. This same reasoning applies to sighted or blind policy interventions. So, no outcomes that can be implemented via an incentive compatible mechanism could dominate the outcomes we consider here.
6 Concluding Remarks

We have shown that, when identity is fully visible and contractible, the efficient affirmative action policy avoids explicit human capital subsidies for the disadvantaged. This seemingly counter-intuitive result follows immediately, once the problem has been placed within an optimal taxation framework. To prefer a group at the assignment stage of the production process is already to implicitly subsidize their acquisition of skills. If these implicit benefits are correctly anticipated by the agents, and if the ex post intervention is itself efficient, then no further interference with investment incentives is desirable.

This is no longer the case when preferential policies must be identity-neutral. Even with an efficient ex post policy, private and social returns from ex ante investment will generally not coincide under blindness. We have shown that the second-best, identity-neutral intervention to increase opportunity for a disadvantaged group involves setting a lower productivity standard for assigning agents to slots than would occur under laissez-faire. Moreover, we have derived an empirically meaningful condition under which the efficient blind policy entails a general subsidy to human capital investments.
References


Figure 1: Sequence of Actions
Figure 2: Competitive Equilibrium under Laissez-faire

\[ F(\pi, p) = 1 - \theta \]

\[ G^{-1}(\pi) = \int_{\Delta F(v)}^{\infty} dv \]
Figure 3: Optimal Sighted Affirmative Action